

XXXIII. *An Account of the Calculations made from the Survey and Measures taken at Schehallien, in order to ascertain the mean Density of the Earth.* By Charles Hutton, Esq. F. R. S.

Read May 21, 1778. **T**HE survey from which these calculations have been made was taken at and about the hill Schehallien in Perthshire, in the years 1774, 1775, and 1776, by the direction, and partly under the inspection, of the Rev. NEVIL MASKELYNE, D. D. F. R. S. and Astronomer Royal, by whom the manner of making the survey has already been fully explained in the Philosophical Transactions for 1775.

I have therefore only to give an account of the measures of the lines and angles, and of the calculations which I have raised from them with all possible care and faithfulness, for the purpose of determining the measure of the ratio of the mean density of the earth to that of water or any other known matter.

These calculations were naturally and unavoidably long and tedious; and the more so as the business was in a manner quite new, which laid me under the necessity

of inventing and describing such modes of computation as should be proper to be applied in so important and delicate a business. Having, at length, with close and unwearied application for a considerable time completed all the calculations; I have, in the following sheets, drawn up an account of those operations, with the results arising from them; and have accompanied them with such drawings as are necessary to illustrate the descriptions. I have also inserted a synopsis of the measures which were taken of the lines and angles, from which any person may at any time satisfy himself of the truth of the computations that have been made and are herein described. These measures I have here immediately subjoined, before I proceed to describe the computations made from them..

A synopsis of the horizontal and vertical angles that were observed at the principal points in making the survey about Schehallien.

In the first column are contained the names of the horizontal angles, the measure of which, in degrees and minutes, are in the second column, and the vertical angles are in the third column, in which it is to be observed, that the letter denoting the object is placed before the degrees and minutes, and E or D after them, to shew that they are in elevation or depression respectively. The mark * placed to the measure of any angle denotes that it is the mean of the two observations made with the instrument turned different ways; namely, after the first observation, reversing it to make the second. Also the mean height of the theodolite is put down to each station. In the vertical angles, the bottom of the object is understood, unless where the top is mentioned below, and sometimes the height of the pole is added in feet and inches,

At A	Theodolite = 4 ft. 10 in.		At F	Theodolite = 4 ft. 8½ in.	
DAB	31 15	° ' "	GFW	17 27	W 0 58 D
DAN	77 30	N 12 22E	GFK	76 7½	K 9 50 E
DAO	102 36½	O O 58E	GFN	103 57½	N 17 42½E
DAR	134 31½		GFD	172 54	D 3 37 E
NAC	83 13½	N 12 20E	HFG	10 9	G 1 36 D
NAO	25 4		HFP	60 41	P 5 9 E
OAS	27 24	S 5 25D	HFK	65 58	K 9 41 E
OAR	31 55	R 6 31D	HFN	93 49	N 17 35½E
At B			At G	Theodolite = 4 ft. 6½ in.	
DBN	81 8		DGF	6 8	
DBO	101 41		NGD	59 41	
DEA	139 59		NGF	65 49	
At C	Theodolite = 4 ft. 7½ in.		NGH	101 9	
ACD	126 6		HGF	166 57	
ACF	93 34		HGD	160 49	
ACG	92 15		HGX	71 11½	
ACH	85 31½		HGW	4 32	
ACN	49 10	N 10 4E	PGF	95 56	F 1 15 E
ACB	8 11		PGN	30 6½	N 16 54 E top of Cairn.
At D	Theodolite = 4 ft. 8½ in.		PGK	0 9½	K 10 15 E
ADC	48 7		PGL	62 57	L 0 32 D
ADB	8 45		PGW	63 20	W 0 54 D
ADN	49 14	N 10 40E	PGH	71 2	H 3 0 D
ADW	83 56½	N 10 39E			P 5 46 E top of ball.
ADH	88 56	A 1 15E			
ADG	95 5	G 3 17D			
ADF	96 3	F 3 34D			

At H	Theodolite = 4 ft. 5½ in.		WLN	38 58	N 10° 56½ E
GHF	2 54	G 2 59 E		38 58½ *	
GHD	13 2		WLK	60 33	K 10° 24½ E
NHG	49 44	N 14 42 E		60 34 *	
NHK	29 17		WLT	115 38 *	T 0° 30 D
NHW	107 43	N 14 39 E top	WLV	126 58½ *	V 0° 35½ D
NHW	107 41		WLY	140 50½ *	Y 2° 35 D
KHD	65 59	K 12 6 E top	WLF	3 58½ *	F 0° 38½ E
KHG	79 1		WLE	62 12 *	E 10° 16½ E
KHW	78 25	W 3 2 E	WLi	38 4½ *	i 4° 34½ E
WHP	96 14	P 7 39 E top			
WHp	149 2	p 0 22 E			
WHG	157 25½	G 2 55 E			
At w	Theodolite = 4 ft. 5½ in.		At Y	Theodolite = 4 ft. 8 in.	
LWK	107 27½	L 0 25½ E	TYK	69 23½ *	K 10° 29 E
LWP	130 40	P 5 43 E top	TYN	76 56½ *	N 9° 37½ E
LWP	130 41		TYL	124 47 *	L 2° 35 E
LWN	133 48½	N 12 36½ E	TYG	103 24½ *	G 1° 36½ E
LWF	175 2	K 11 30 E	TYZ	19 21½ *	Z 3° 3½ E
LWQ	4 41		TYV	13 29½ *	V 3° 26½ E
LWG	178 21		VYT	13 28 *	T 2° 4½ E
LWH	193 13	H 3 14 D	VYZ	32 52 *	Z 3° 3½ E
At L	Theodolite = 4 ft. 9 in.		VYE	81 49½ *	E 10° 33 E
GLP	37 1	P 4 24½ E	VYK	82 53 *	K 10° 29 E
GLY	139 32½ *	Y 2 35½ D	VYN	90 24 *	N 9° 36½ E
WLG	1 18	W 0° 40½ D	VYG	116 52 *	G 1° 35½ E
	1 17½ *	G 0° 31 E	VYH	126 57 *	H 1° 0 E
WLP	38 12	P 4° 22½ E	VYW	130 33 *	W 2° 2 E
	38 12½ *		VYL	138 14 *	L 2° 33½ E

At v	Theodolite = 4 ft. 11 in.		UZY	117 21 *	Y 3 * 8 $\frac{1}{2}$ D
TVZ	17 25 $\frac{1}{4}$ *	Z 2 * 28 $\frac{1}{2}$ E	UZV	134 4 $\frac{1}{2}$ *	V 2 * 31 $\frac{1}{2}$ D
	17 25 *	2 * 26	UZT	144 27 $\frac{1}{2}$ *	T 3 * 56 $\frac{1}{4}$ D
TVU	34 24 $\frac{1}{2}$ *	U 2 * 47 $\frac{1}{2}$ E			U 2 * 45 $\frac{1}{2}$ E
TVF	70 43 *	E 9 * 10 E	At u	Theodolite = 4 ft. 10 in.	
TVK	71 23 $\frac{1}{2}$ *	K 9 * 5 $\frac{1}{2}$ E	ZUT	18 20 $\frac{1}{4}$ *	T 3 * 32 $\frac{1}{4}$ D
	71 24 *	9 * 5 $\frac{1}{2}$	ZUV	28 54 $\frac{1}{2}$ *	V 2 * 51 D
TVW	112 32 $\frac{1}{2}$ *	W 0 * 25 $\frac{1}{2}$ E	ZUO	124 42 *	O 4 * 1 $\frac{1}{2}$ E
TVL	119 57 *	L 0 * 34 E	ZUN	125 9 $\frac{1}{2}$ *	N 10 * 13 $\frac{1}{2}$ E
TVY	147 51 $\frac{1}{2}$ *	Y 3 * 31 $\frac{1}{2}$ D	ZUX	128 2 $\frac{1}{4}$ *	X 10 * 55 $\frac{1}{2}$ D
	147 50 *	3 * 28 $\frac{1}{2}$ D	ZUA	157 55 *	A 1 * 36 $\frac{1}{4}$ E
		T O 6 E	ZUR	160 37 $\frac{1}{2}$ *	R 3 * 38 D
At T	Theodolite = 4 ft. 10 in.				Z 2 * 51 D
YTV	18 41 $\frac{1}{2}$ *	V 0 * 8 $\frac{1}{2}$ D	At x	Theodolite = 4 ft. 7 in.	
YTL	30 1 *	L 0 * 27 $\frac{1}{4}$ E	OXA	40 59 $\frac{1}{2}$ *	A 4 * 15 E
YTU	116 18 $\frac{1}{4}$ *	U 3 * 31 E	OXS	53 20 *	S 0 * 44 $\frac{1}{4}$ E
Y TZ	133 31 $\frac{1}{2}$ *	Z 3 * 49 $\frac{1}{2}$ E	OKZ	139 22 $\frac{1}{2}$ *	U 10 * 49 $\frac{1}{4}$ E
		Y 2 10 D	OXU	174 59 *	Z 1 * 48 $\frac{1}{4}$ E
VTY	18 41 *	Y 2 * 6 D			O 11 * 13 $\frac{1}{2}$ E
VTL	48 43 *	L 0 * 30 E	At s	Theodolite = 4 ft. 9 in.	
VTE	94 2 *	E 9 * 42 E	RSA	18 27	A 5 24 $\frac{1}{4}$ E
VTU	135 1 $\frac{1}{2}$ *	U 3 * 31 $\frac{1}{2}$ E	RSA	18 28	A 5 20 $\frac{1}{4}$ E
VTZ	152 14 $\frac{1}{2}$ *	Z 3 * 50 E	RSN	72 51	N 18 40 E
		V O 7 D	RSO	90 58	O 13 19 $\frac{1}{2}$ E
At z	Theodolite = 4 ft. 6 $\frac{1}{2}$ in.		RSX	171 1	X 0 55 $\frac{1}{2}$ D
UZR	10 53 $\frac{1}{2}$ *	R 0 * 50 D			R 0 24 $\frac{1}{2}$ E
UZA	16 5 *	A 1 * 58 $\frac{1}{4}$ E			
UZX	16 18 *	X 1 * 52 D			
UZO	33 18 $\frac{1}{4}$ *	O 3 * 58 E			

At R the E. end of the S. bale.	Theodolite = 4 ft. 6 in.		XOZ	$23 \ 38\frac{1}{2}^*$	Z $0^* \ 0^* \ D$
KRN	41 $6\frac{1}{2}$		MOR	16 $36\frac{1}{2}^*$	X $11^* \ 18^* \ D$
B''RA	20 $36\frac{1}{4}^*$	A $6^* \ 27\frac{1}{2}^* \ E$	MOB''	47 23^*	R $12^* \ 3\frac{1}{2}^* \ D$
B''RN	84 $32\frac{3}{4}^*$	N $19^* \ 17\frac{1}{2}^* \ E$	MOA	73 $32\frac{1}{2}^*$	B'' $7^* \ 39\frac{1}{2}^* \ D$
B''RO	111 $46\frac{1}{4}^*$	O $11^* \ 57\frac{1}{4}^* \ E$	MOB	81 22^*	A $1^* \ 0\frac{1}{2}^* \ D$
B''R α'	100 $14\frac{1}{2}^*$	α' $20^* \ 2^* \ E$	MON	134 39	B $1^* \ 1\frac{1}{2}^* \ D \ top$
B''R β'	121 $27\frac{1}{4}^*$	β' $15^* \ 11\frac{1}{4}^* \ E$	NOK	95 1	M $13 \ 16$
B''RS	177 39		MO α'	156 34^*	α' $33^* \ 47\frac{1}{2}^* \ E \ top$
B''RZ	186 $53\frac{1}{2}^*$	Z $0^* \ 46^* \ E$	MO β'	125 45^*	β' $21^* \ 29\frac{1}{2}^* \ E$
its sup.	173 $6\frac{1}{2}^*$	O $12 \ 29^* \ E$	MOZ	65 $51\frac{1}{2}^*$	Z $4^* \ 0^* \ D$
At B'' the W. end of the S. bale.	Theodolite = 4 ft. 10 in.		MOU	43 $51\frac{1}{2}^*$	U $4^* \ 3\frac{1}{2}^* \ D$
RB''s	O $55\frac{1}{2}$	R $0 \ 37^* \ D$	MOX	42 $13\frac{1}{2}^*$	X $11^* \ 19^* \ D$
RB'' β'	36 $8\frac{1}{4}^*$	β' $10^* \ 24^* \ E$	MOS	6 $34\frac{1}{2}^*$	S $13^* \ 22\frac{1}{2}^* \ D$
RB''O	37 27^*	O $7^* \ 35\frac{1}{4}^* \ E$	m is in the meridian passing through o, and is a little South of the intersection of that meridian and the line as.		
RB'' α'	50 $49\frac{1}{2}^*$	α' $15^* \ 48^* \ E$	At P the N. ob- servatory.	Theodolite = 4 ft. 10 in.	
RB''N	65 $46\frac{1}{4}^*$	N $17^* \ 33^* \ E$	NPK	77 $18\frac{1}{4}$	
RB''A	139 $2\frac{1}{4}^*$	A $11^* \ 22\frac{1}{4}^* \ E$	GPK	179 $35\frac{1}{4}^*$	G $5 \ 39\frac{1}{4}^* \ D$
At o the S. ob- servatory.	Theodolite = 4 ft. 7 in.		GPH	47 $45\frac{1}{4}^*$	H $7^* \ 32\frac{1}{2}^* \ D$
XOS	35 39^*	S $13^* \ 25\frac{1}{2}^* \ D$	GPW	69 $0\frac{3}{4}^*$	W $5^* \ 37^* \ D$
XOM	42 $12\frac{3}{4}^*$	M $13^* \ 12\frac{1}{2}^* \ D$	GPL	80 $9\frac{1}{4}^*$	L $4^* \ 27\frac{1}{4}^* \ D$
XOR	58 $49\frac{1}{4}^*$	R $12^* \ 4^* \ D$	GPF'	20 54^*	F' $5^* \ 6^* \ D$
XOB''	89 35^*	B'' $7^* \ 39\frac{1}{2}^* \ D$	GPM	38 $18\frac{1}{4}^*$	m $12^* \ 20^* \ D$
XOA	115 $43\frac{1}{2}^*$	A $0^* \ 59^* \ D$	$\gamma'P\delta'$	45 39	} by reduction
XOU	1 39^*	U $4^* \ 4\frac{1}{2}^* \ D$	$\gamma'PK$	2 $55\frac{1}{4}$	

γ' from P bears $30^\circ \ 41' \ 2'' \ E.$ of South.
 δ' from P bears $14 \ 57' \ W.$ of South.
m is Mr. MASON's mark.

At N the West- ern cairn.	Theodolite = 4 ft. 11 in.		At K the East- ern cairn.	Theodolite = 4 ft. 8 in.	
KNO	40 5 $\frac{1}{2}$	} by reduction	NKO	44 51 $\frac{1}{4}$	} by reduction
KNR	74 48 $\frac{2}{3}$		NKR	64 5 $\frac{1}{2}$	
KNP	50 47 $\frac{2}{3}$		NKP	51 54 $\frac{1}{2}$	
KN δ '	9 37 $\frac{1}{2}$		NK α	0 45 *	
KN γ '	2 11 $\frac{2}{3}$		NKF	43 32 $\frac{1}{2}$ *	N 6 * 41 E
KNA	133 53 *	A 12 * 22 $\frac{1}{2}$ D	NKG	51 40 *	n 6 * 36 E top
KNB	146 15 $\frac{1}{2}$ *	B 11 * 17 D top			F 9 * 46 D top
KNC	178 30 $\frac{1}{2}$ *	C 10 * 4 D			Pole 4 ft. 2 in.
KND	172 51 *	D 10 * 39 D top			G 10 * 16 D top
KN η	144 37 $\frac{1}{2}$ *	η 4 * 30 $\frac{1}{2}$ D	NKH	55 12	Pole 4 ft. 4 in.
KNG	98 23 *	G 16 * 55 D top			G 10 * 14 $\frac{1}{2}$ D
		Pole 4 ft. 4 in.	NK α	81 28	α 10 3 D
KNH	69 13 $\frac{1}{2}$	H 14 38 D top	NKH	81 27 $\frac{1}{2}$ *	H 12 7 D top
		Pole 6 ft. 5 in.	NKW	97 18 *	Pole 6 ft. 5 in.
KN μ	64 27 *	μ 18 * 55 D			H 12 * 5 $\frac{1}{2}$ D
KNW	56 22 *	W 12 * 35 D top	NKL	109 19 $\frac{3}{4}$ *	w 11 * 28 D top
		Pole 7 ft.			Pole 7 ft.
KNL	49 7 *	L 10 * 55 D top	NKL	109 18 $\frac{1}{2}$ *	L 10 * 25 D top
		Pole 6 ft. 8 in.			Pole 6 ft. 8 in.
KNY	18 50 *	Y 9 * 37 $\frac{1}{2}$ D top	NKY	153 40 *	L 10 * 23 $\frac{1}{2}$ D
KN γ '	2 15 *	γ ' 7 * 30 D	NKV	174 20 *	Y 10 * 30 D top
KNE	1 15 $\frac{1}{2}$ *	E 6 * 38 $\frac{1}{2}$ D	NKE	156 18	V 9 * 5 $\frac{1}{2}$ D top
KN δ '	9 37 $\frac{1}{2}$ *	δ ' 6 * 25 $\frac{1}{2}$ D	NK α '	11 36 *	E 3 34 D top
KNU	41 4 $\frac{1}{2}$ *	U 10 * 12 $\frac{1}{2}$ D top	NKM'	7 8 *	α ' 6 * 31 $\frac{3}{4}$ E
			NK δ '	6 2 $\frac{2}{3}$ *	M' 4 * 51 $\frac{1}{4}$ E
KNS	60 44 *	S 18 * 41 D			δ ' 6 * 53 E
KNR	74 48 $\frac{1}{2}$ *	R 19 * 20 D	NK α	54 26 $\frac{1}{2}$ *	α 10 3 $\frac{1}{2}$ D
KNB''	104 30 *	B' 17 * 35 $\frac{1}{2}$ D	NK α	54 25 $\frac{1}{2}$ *	α 10 * 3 $\frac{1}{2}$ D
		K 6 * 43 $\frac{1}{2}$ D top	NK β	39 16 $\frac{1}{2}$ *	β 8 * 58 $\frac{1}{2}$ D
			α' K β '	37 39	α' 6 32 E

At

At α the E. end of the N. baf.	Theodolite = 4 ft. 4 in. o /	o /	At n the new W. cairn.	Theodolite = 4 ft. 9 in. o /	o /
$\gamma \alpha K$	106 0 *	K 10 * 1 $\frac{1}{2}$ E	$K n \alpha$	108 59 $\frac{1}{2}$ *	K 6 * 40 $\frac{1}{2}$ D
$\gamma \alpha G$	101 3 *	G 9 * 37 $\frac{1}{2}$ E	$K n \beta$	128 35 $\frac{1}{2}$ *	β 13 * 7 $\frac{1}{2}$ D
$\gamma \alpha N$	89 41 *	N 13 * 45 $\frac{1}{2}$ E	$K n F'$	128 55 $\frac{1}{2}$ *	F' 14 * 10 $\frac{1}{2}$ D
$\gamma \alpha n$	89 25 *	n 13 * 49 $\frac{1}{2}$ E	$K n \gamma$	135 40 $\frac{1}{2}$ *	γ 12 * 29 D
$\gamma \alpha F'$	47 38 *	F' 8 * 6 $\frac{1}{2}$ E	$K n D$	173 55	D 10 * 45 D
		β 0 19 D	$K n N$	38 28 $\frac{1}{2}$ *	N 1 42 D
		γ 0 0 top	$K n B''$	102 54 $\frac{1}{4}$ *	B'' 17 31 D
		Pole 3 ft. 2 in.	$D n A$	53 49 *	A 12 * 21 $\frac{1}{2}$ D
			$D n G$	75 24 *	G 17 * 3 $\frac{1}{2}$ D
			$D n F''$	78 44 $\frac{1}{4}$	F'' 20 * 2 $\frac{1}{2}$ D
					p'' is a pole in a line with F and K
At β between α and γ the ends of the N. baf.	Theodolite = 4 ft. 8 in.		$D n H$	104 35	H 14 * 42 $\frac{1}{2}$ D
$\alpha \beta k$	174 51	k 0 41 $\frac{1}{2}$ E	$D n L$	124 34	L 10 56 D
$\alpha \beta i'$	147 37 $\frac{1}{2}$	i' 1 * 25 $\frac{1}{2}$ E	$D n K$	173 55	α 13 * 51 $\frac{3}{8}$ D
$\alpha \beta F'$	71 49 *	F' 10 * 22 $\frac{3}{4}$ E	$N n G$	132 59	α 13 * 54 D
$\alpha \beta D$	108 1 $\frac{1}{4}$ *	D 9 * 17 $\frac{1}{2}$ E			N n was = 93 $\frac{1}{2}$ feet by the tape measure.
$\alpha \beta n$	70 58 $\frac{1}{2}$ *	n 13 * 5 $\frac{3}{4}$ E			
$\gamma \beta D$	71 57 $\frac{3}{4}$ *	D 9 * 17 $\frac{1}{6}$ E			
$\gamma \beta F'$	108 9 $\frac{1}{2}$ *	F' 10 * 22 $\frac{3}{4}$ E			
		α 0 * 6 D			
At γ the West- ern end of the North baf.	Theodolite = 4 ft. 8 $\frac{1}{2}$ in.		At E the new E. cairn.	Theodolite = 4 ft. 9 in.	
$\alpha \gamma F'$	50 6 $\frac{1}{2}$ *	F' 8 * 23 $\frac{1}{2}$ E	$N E A$	10 5 $\frac{1}{2}$ *	A 6 * 22 $\frac{1}{2}$ E
$\alpha \gamma n$	63 53 $\frac{1}{2}$ *	n 12 * 27 $\frac{1}{4}$ E	$N E M'$	4 58 $\frac{1}{2}$ *	M' 4 * 45 $\frac{1}{2}$ E
$\alpha \gamma D$	97 5 $\frac{3}{4}$ *	D 9 * 41 E	$N E D$	4 51 $\frac{1}{2}$ *	D 6 * 41 $\frac{1}{2}$ E
		α 0 * 2 D top	$N E K$	22 14 $\frac{1}{2}$ *	K 2 * 11 $\frac{1}{2}$ E
		Pole 3 ft. 2 in.	$N E H$	78 40 $\frac{1}{2}$ *	H 11 * 47 $\frac{1}{2}$ D top
			$N E W$	94 17 *	Pole 6 ft. 5 in.
			$N E L$	106 21 $\frac{1}{2}$ *	W 11 * 16 $\frac{1}{2}$ D top
					Pole 7 ft.
					L 10 * 17 D top
					Pole 6 ft. 8 in.

			At the center of the transit instrument near the N. observ.	Theodolite = 4 ft. 8½ in.	
NEY	151 16 *	Y 10 * 37 D			
NEV	172 20 *	V 9 * 15 D			
NBT	172 22½ *	T 9 * 41½ D top			
NEZ	144 46 *	Z 9 * 18½ D	M''tP'	32 24 *	P' 9 * 18½ D
NEU	114 20½ *	U 10 * 4½ D	M''tG	32 57½ *	G 5 * 58½ D
a'Ea'	68 35½ *	a' 6 * 21 E	M''tm	5 14 *	m 12 * 32½ D
a'Ed'	73 50½ *	d' 6 * 37½ E	M''tH	14 28 *	H 7 * 44½ D
a'EN	78 39½ *	N 6 * 33 E	M''tW	35 23½ *	W 5 * 47½ D
a'Eb'	97 17 *	b' 9 * 5½ D	M''tL	46 22 *	L 4 * 35½ D
		a' 8 * 27 D	mtP	68 13 *	p 12 * 50½ E
			mtM'	68 15 *	M' 22 * 5 E
At M' the meridian mark on the top of the hill South off.	Theodolite = 3 ft. 10½ in.		mtG	144 17 *	G 5 * 57½ D
KM'E	3 28½ *	E 4 * 56½ D	mtH	96 52 *	H 7 * 43½ D
KM'γ'	6 37½ *	γ' 6 * 3½ D top	mtW	75 56 *	W 5 * 47½ D
KM'L	53 27½ *	L 10 * 19 D top Pole 6 ft. 8 in.	mtL	64 58 *	L 4 * 37½ D
KM'W	63 12½ *	W 11 * 41 D top Pole 7 ft.	mtP	54 4½ *	P 17 * 42½ D top
KM'H	78 4½ *	H 12 * 56½ D top Pole 6 ft. 5 in.	M'' bears 1' 8" W. of North. M' bears South. P' in a line with K and P. P, a pole immediately above or South of the transit instrument.		
KM'm	86 33½ *	m 22 * 12 D	At p	Theodolite = 4 ft. 5 in.	
KM'p	87 54 *	p 22 * 10 D	GpM'	147 5 *	M' 22 * 8½ E
KM'G	108 49 *	G 12 * 33 D top Pole 4 ft. 4 in.	Gpγ'	133 6½ *	γ' 24 * 18 E
KM'F	118 3½ *	F 12 * 25 D top Pole 4 ft. 2 in.	GpF	26 43	F 5 26 D
KM'δ	176 1½ *	δ 12 * 8 E	GpP'	0 34½ *	P' 9 * 23½ D
		K 5 5 D	GpM''	32 47¾ *	M'' 13 * 25½ D
			Gpt	32 48	t 19 17 D
			Gpm	38 0½ *	m 12 * 34½ D
			GpH	47 11 *	H 7 * 49½ D

GpW	68 3½ *	W 5 * 51½ D	Ka'o	73 39 *	o 33 * 52 D
GpP	76 55½ *	P 13 * 49½ D	Ka'β'	28 4½ *	β' 15 * 56½ D top
GpL	79 0½ *	L 4 * 39½ D	Ka'γ'	4 53½ *	γ' 7 * 35½ top
Gpm'	121 24 *	m' 4 * 50½ D	Ka'δ	65 24 *	δ' 3 41½ top Pole 17 ft. 4 in.
GpE	179 37 *	E 17 * 39½ E	Ea'd'	13 48 *	d' 12 * 54½ D
m' is a pole a little above or South of P.			Ea'a'	8 36½ *	a' 8 * 3½ D
			Ea'K	2 49½ *	E 6 30 D
At m'			At β'	Theodolite = 4 ft. 9 in.	
Gm'M'	145 4½ *	M' 22 * 8½ E	the South-east pole.		
Gm'δ	131 11 *	δ' 24 * 12½ E			
Gm'p	57 53½ *	p 2 * 47 D			
Gm't	34 43	t 10 15 D			
Gm'F	13 1 *	F 5 * 22½ D	α'β'o	56 47 *	o 21 * 45½ D
Gm'P'	0 29½ *	P' 19 * 19 D	α'β'E'	62 21½ *	B' 10 * 28½ D
Gm'M''	32 29½ *	M'' 13 * 23½ D	α'β'R	84 46 *	R 15 * 17 D
Gm'P	37 6 *	P 18 * 16½ D	α'β'M	96 41 *	M 17 * 3½ D
Gm'm	37 49½ *	m 12 * 36½ D	α'β'K	114 8½ *	α' 15 * 42½ E
Gm'H	47 10 *	H 7 * 49 D			
Gm'W	68 11 *	W 5 * 49½ D	At γ'	Theodolite = 4 ft. 4 in.	
Gm'L	79 17½ *	L 4 * 40½ D	the North-east pole.		
Gm'E	177 53 *	E 17 * 54 E			
At m			Nγ'P	56 55½ *	P 18 * 3½ D
PmK	15 57	K 15 0 E	Nγ't	55 22½ *	t 17 49½ D
Pma	37 11	P 12 27 E	Nγ'n	0 50½ *	n 7 15 E
At α'	Theodolite = 4 ft. 11 in.		Nγ'δ	8 17½ *	δ' 7 * 59½ E
the South-west pole.			Nγ'a'	14 17 *	a' 7 * 31 E
Kα'N	149 3 *	K 6 * 43½ D	Nγ'K	151 54½ *	K 1 * 34½ E
Kα'B'	131 3 *	B' 15 * 51½ D			N 7 * 23½ E
Kα'R	102 7½ *	R 20 * 9 D			

At a'	Theodolite = 4 ft. 10 in.	At c'	Theodolite = 4 ft. $8\frac{1}{2}$ in.		
$Ea'a'$	$102^{\circ} 48' *$	$a' 7^{\circ} 56' E$	$121^{\circ} 12\frac{1}{2}' *$	$b' 6^{\circ} 1' E$	
$Ea'd$	$97^{\circ} 5\frac{1}{2}' *$	$d' 8^{\circ} 10' E$		$a' 9^{\circ} 6' E$	
$Ea'b'$	$65^{\circ} 58\frac{1}{2}' *$	$b' 6^{\circ} 4\frac{1}{2}' D$			
$Ea'c'$	$88^{\circ} 6' *$	$c' 11^{\circ} 20\frac{1}{2}' D$	At d'	Theodolite = 4 ft. 5 in.	
$Ea'd'$	$108^{\circ} 21\frac{1}{2}' *$	$d' 2^{\circ} 34\frac{1}{2}' E$	$a'd'a'$	$169^{\circ} 14\frac{1}{2}' *$	$a' 12^{\circ} 35\frac{1}{2}' E$
		$E 7^{\circ} 19' E$			$a' 2^{\circ} 48' D$

At b'	Theodolite = 4 ft. $9\frac{1}{2}$ in.	
$c'b'a'$	$36^{\circ} 41\frac{1}{2}' *$	$a' 5^{\circ} 44\frac{1}{2}' E$
$c'b'E$	$53^{\circ} 26\frac{1}{2}' *$	$E 8^{\circ} 41\frac{1}{2}' F$
		$c' 4^{\circ} 41' D$

Several other angles and bearing of objects were taken, which, being of no use in computing the attraction of the hill, are here omitted.

The foregoing tables, containing all the angles collected together which were observed at the same point, include all the horizontal angles that were at different times taken for ascertaining the relative places of the principal points and objects on a horizontal plane. The numerous other angles used, in finding the sections of the ground, are given hereafter, with their computed results annexed to them.

We now proceed to speak of the two principal bases which were accurately measured, as foundations on which every thing else must depend; and first,

Of

Of the measure of the base RB" in Glenmore, the valley on the South of Schehallien, taken the 16th, &c. of Sept. 1774.

Here A and B are the names of the two measuring rods, which were laid down alternately in the order as expressed in the following table of measures. The lengths of these rods, by the brass standard, when the thermometer was at $62\frac{3}{4}$, were thus, *viz.*

$$\left. \begin{array}{l} A = 20 \text{ feet } 1' 255 \text{ inch.} = 20 \cdot 10458 \\ B = 20 \text{ feet } 1' 323 \text{ inch.} = 20 \cdot 11025 \end{array} \right\} \text{ feet.}$$

The numbers following each rod, with the sign + interposed, are inches and decimal parts; and they denote the distance beyond the end of each rod to the beginning of the next following rod; and, therefore, the sum of all these numbers must be added to the sum of the lengths of the rods themselves for the total of the measures. Also, as the first rod began at 2 feet 8 inches from the point R, this number is to be added to the total last mentioned, to give the measure of the whole base from R to B".

$$A + 8 \cdot 29$$

A +8.29	B +3.45	B +7.22	A +3.34	A +4.47	A +3.69
B +2.53	A +3.80	A +1.91	B +3.65	B +3.75	B +4.07
A +6.11	B +6.64	B +4.46	A +6.96	A +4.74	A +2.75
B +6.66	A +7.76	A +1.95	B +3.07	B +3.06	B +4.65
A +2.79	B +3.28	B +2.26	A +3.55	A +2.58	A +4.07
B +1.20	A +4.87	A +4.54	B +2.93	B +4.15	B +3.23
A +2.07	B +6.18	B +4.48	A +5.53	A +3.16	A +4.16
B +4.80	A +8.70	A +3.14	B +5.33	B +4.64	B +5.73
A +0.00	A +7.87	B +3.38	A +4.38	A +3.26	A +4.12
B +1.78	B +4.75	A +5.00	B +3.67	B +4.18	B +4.91
A +3.29	A +6.56	B +4.85	A +5.12	A +4.04	A +3.18
B +2.85	B +5.24	A +6.12	B +1.06	B +2.92	B +3.91
A +6.39	A +7.90	B +3.44	A +5.96	A +3.10	A +5.28
B +4.86	B +6.32	A +6.18	B +2.47	B +5.11	B +2.90
A +6.08	A +6.92	B +4.19	A +3.84	A +4.61	A +4.39*
B +8.58	B +7.28	A +4.51	B +5.57	B +3.34	B +4.37
A +9.07	A +6.34	B +3.04	A +2.63	A +2.57	A +3.29
B +1.53	B +8.93	A +4.37	B +7.41	B +5.80	B +2.12
A +2.28	A +5.39	B +2.96	A +3.11	A +3.37	A +2.95
B +7.47	B +5.20	A +2.47	B +1.74	B +2.58	B +3.30
A +2.40	A +3.54	B +3.90	A +2.07	A +2.24	A +2.82
B +5.42	B +1.26	A +5.78	B +4.33	B +3.48	B +3.97
A +7.42	A +3.20	B +3.97	A +5.93	A +2.95	A +1.37
B +8.14	B +5.34	A +4.87	B +6.36	B +2.88	B +0.00
A +8.77	A +3.74	B +4.83			

The sum of all these is $74A + 73B + 669.28$ inches,

or $74A + 73B + 55.773$ feet, including the 2 feet 8 inches at the beginning of the measurement.

Now 74A is = 1487.73892

73B is = 1468.04825

The odd parts 55.773

Sum 3011.56017 = the base unreduced.

But a reduction of this must be here made according to the state of the thermometer, and for the wearing of the brass 5 feet standard (see Phil. Trans. vol. LVIII. for the year 1768, p. 313, &c.). Now the difference between 62° and $62\frac{3}{4}$ being $\frac{3}{4}$, therefore $3011.56 \times \frac{232}{180000 \times 12} \times \frac{3}{4} = 3011.56 \times \frac{29}{270000} \times \frac{3}{4} = 0.024$ feet, is the small correction on account of the thermometer, and which being added makes the number become 3011.584 for the length of the base as reduced to the state of 62° of FAHRENHEIT's thermometer. But the brass rod had been $\frac{1}{1000}$ th of an inch shortened by wearing, and it was originally $\frac{1}{1000}$ th of an inch shorter than the Royal Society's brass standard yard, so that it is now $\frac{1}{500}$ th inch shorter than that standard in the length of 3 feet, or $\frac{1}{18000}$ th part of the whole; therefore subtracting the $\frac{1}{18000}$ th part, or .167 from the above quantity, there remains 3011.417 feet for the corrected measure of this base, or the true length of the line RB".

The above measures, as far as to that marked * inclusively, together with 10 feet $10\frac{1}{2}$ inches more, reach

to a place to which they had before measured with the tape line, and by it found to be 2844·8 feet; while the measure of the same by the rods is found to be 2839·3 feet. The difference is $5\frac{1}{2}$ feet, a small part of which might be owing to the unstable state of the wooden stands used in the first quarter of the base; but the greater part of this difference is more likely to be owing to the uncertain way of measuring with a tape, which, to say nothing of the ground not being quite level, is liable to be stretched more or less in length with different degrees of tension, and to be variously warped in length by moisture.

Of the measurement of the base $\alpha\beta\gamma$ in Rannoch, to the North-west of the bill of Schehallien.

1. One part of this base was measured twice over in different ways. The part $\alpha\beta$ was carefully measured on the 8th of October 1774 with a chain, and found to be 63 chains and $40\frac{1}{2}$ links, or 63·405 chains in length.

Now on the 24th of the same month the chain was measured by means of the five-foot brass standard, when the thermometer was at $38^{\circ}\frac{1}{2}$, and the length found to be 65·94542 feet. Hence then $65\cdot94542 \times 63\cdot405 = 4181\cdot269$ is the length of all the chains, to which, add-

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ing 1.764 the breadth of the 63 iron pins, the sum is 4183.033 for the length of $\alpha\beta$ uncorrected.

But $62 - 38\frac{1}{2} = 23\frac{1}{2}$, therefore $-23\frac{1}{2} \times \frac{29}{2700000} \times 4183 = -1.056$ is the reduction on account of the state of the thermometer, which being applied with its proper sign, there results 4181.977; and from this last number deducting again $\frac{1}{18000}$ th part or .232, on account of the wearing of the brass standard, there then remains 4181.745 feet for the length of the part $\alpha\beta$ of the base in Rannoch, as measured by the chain.

But as the chain was measured not at the same time with the base, but between two and three weeks later, when the air was probably cooler, the reduction above made for the state of the thermometer is perhaps something too great, and we may safely conclude $\alpha\beta$ to be equal 4182 feet as measured by the chain.

2. The whole base $\alpha\beta\gamma$ was next, on the 10th, 11th, and 12th of October, very carefully measured by the twenty-foot measuring rods. The rods at that time measured thus, $\left\{ \begin{array}{l} A = 20 \text{ ft.} + 1.306 \text{ inch.} = 20.108\frac{5}{6} \\ B = 20 \text{ ft.} + 1.354 \text{ inch.} = 20.112\frac{5}{6} \end{array} \right\}$ feet; the thermometer being then at 40° . The number of rods and the additional parts were as follows.

A +4.49	A +3.99	B +1.73	B +3.22	B +2.43	B +1.87
B +3.29	B +3.13	A +2.41	A +3.16	A +1.97	A +2.38
A +6.57	A +3.55	B +2.99	B +2.07	B +1.74	B +2.67
B +3.62	B +4.19	A +2.62	A +2.22	A +2.84	A +2.07
A +3.84	A +4.06	B +2.18	B +4.83	B +3.65	B +1.84
B +3.52	B +3.94	A +2.72	A +2.39	A +1.62	A +4.03
A +4.50	A +3.64	B +3.02	B +2.68	B +1.27	B +1.72
B +3.62	B +3.23	A +2.46	A +2.09	A +2.38	A +2.14
A +4.88	A +3.76	B +3.68	B +1.97	B +2.36	B +1.80
B +2.74	B +2.56	A +2.62	A +1.48	A +2.57	A +1.27
A +3.24	A +3.38	B +2.72	B +3.20	B +2.07	B +1.88
B +4.30	B +3.29	A +3.33	A +2.62	A +2.48	A +2.36
A +3.50	A +3.65	B +2.93	B +2.37	B +2.31	B +2.04
B +3.26	B +3.51	A +3.14	A +2.47	A +2.28	A +2.37
A +2.96	A +3.88	B +2.93	B +3.48	B +3.96	B +1.77
B +3.32	B +2.29	A +2.40	A +3.16	A +4.87	A +1.66
A +5.93	A +3.13	B +2.06	B +3.50	B +2.61	B +2.26
B +5.00	B +3.71	A +3.13	A +2.33	A +2.22	A +1.97
A +3.43	A +3.13	B +2.57	B +2.37	B +1.63	B +1.77
B +3.87	B +3.13	A +2.85	A +2.68	A +1.87	A +2.63
A +6.37	A +5.43	B +2.87	B +2.68	B +2.41	B +3.18
B +3.48	B +3.08	A +4.12	A +2.70	A +2.62	A +2.09
A +4.86	A +3.57	B +2.20	B +2.68	B +2.09	B +1.74
B +4.87	B +5.68	A +2.68	A +2.05	A +2.27	A +2.74
A +3.08	A +3.89	B +2.17	B +3.09	B +3.02	B +4.49
B +3.67	B +2.78	A +2.13	A +2.50	A +2.41	A +2.60
A +3.28	A +3.27	B +2.17	B +2.52	B +2.53	B +2.45
B +3.43	B +1.84	A +3.12	A +2.62	A +1.84	A +1.41
A +4.82	A +0.00	B +2.70	B +2.43	B +2.68	B +2.67
B +4.46	A +3.14	A +3.17	A +3.01	A +2.37	A +1.98

B + 2.96	A + 1.66	B + 2.77	B + 2.37	B + 1.86	B + 2.49
A + 2.27	B + 1.53	A + 2.09	A + 2.86	A + 2.33	A + 2.11
B + 2.60	A + 1.80	B + 2.14	B + 2.42	B + 2.65	B + 3.03
A + 3.08	B + 3.53	A + 2.45	A + 2.16	A + 2.14	A + 2.67
B + 2.23	A + 2.45	B + 2.98	B + 2.21	B + 2.35	B + 2.43
A + 4.70	B + 0.00*	A + 2.40	A + 2.75	A + 2.43	A + 1.93
B + 2.28	A + 2.36	B + 2.50	B + 2.03	B + 2.59	B + 2.75
A + 1.79	B + 2.75	A + 2.82	A + 2.73	A + 2.48	A + 1.99
B + 1.83	A + 1.77	B + 2.37	B + 2.14	B + 2.48	B + 2.17
A + 2.74	B + 1.55	A + 2.76	A + 1.57	A + 2.91	A + 1.83
B + 2.48	A + 1.97	B + 2.91	B + 1.77	B + 2.47	B + 1.93
A + 2.23	B + 2.04	A + 2.58	A + 2.21	A + 1.80	A + 1.65
B + 1.66	A + 2.56	B + 2.34	B + 2.11	B + 1.99	B + 1.04
A + 3.08	B + 2.15	A + 2.86	A + 2.99	A + 2.81	A + 1.96
B + 2.31	A + 2.26	B + 2.67	B + 2.06	B + 2.44	B + 2.77
A + 2.20	B + 2.37	A + 2.19	A + 2.36	A + 2.07	A + 2.25
B + 3.06	A + 1.94	B + 2.37	B + 1.77	B + 2.44	B + 1.79
A + 2.36	B + 1.97	A + 2.93	A + 1.66	A + 2.11	A + 0.00
B + 2.24	A + 1.94				

Of the foregoing measures, the sum of all from the beginning to that marked * inclusively, together with 13 feet 2 inches more, brings us to the point β before measured to by the chain. Now to this place, by adding together the measures, there are found to be 103 A and 102 B, and the sum of the parts is 586.71 inches.

Then

$$\text{Then } 103 A = 103 \times 20 \cdot 108 \frac{5}{6} = 2071 \cdot 210$$

$$102 B = 102 \times 20 \cdot 112 \frac{5}{6} = 2051 \cdot 509$$

$$586 \cdot 71 \text{ inches} = 48 \cdot 893$$

$$13 \text{ ft. } 2 \text{ inch.} = 13 \cdot 167$$

$$\text{Hence } \alpha \beta \text{ (unreduced) is } \underline{4184 \cdot 779}$$

But since $62 - 40 = 22$, therefore the reduction for the state of the air is $-22 \times \frac{29}{2700000} \times 4185 = -.989$, which being applied to the above sum, there remains $4183 \cdot 79$ as corresponding to the state of 62° of the thermometer. From this last number deduct its $\frac{1}{18000}$ th part, *viz.* $.232$, and there results $4183 \cdot 558$ for the correct length of the part $\alpha \beta$ as determined by this very accurate method; which is but about a foot and a half more than what it was found to be by the less accurate measure by the chain, which is a nearer approach to an equality than could well be expected.

To determine now the whole length of the base $\alpha \gamma$; by taking the whole sums there are found to be $146 A$ with $144 B$ and $779 \cdot 78$ inches of the odd parts.

$$\text{Then } 146 A = 146 \times 20 \cdot 108 \frac{5}{6} = 2935 \cdot 890$$

$$144 B = 144 \times 20 \cdot 112 \frac{5}{6} = 2896 \cdot 248$$

$$779 \cdot 78 \text{ inches} = 64 \cdot 982$$

$$\text{The sum or } \alpha \gamma \text{ (unreduced) is } \underline{5897 \cdot 120}$$

The correction for the thermometer is $-22 \times \frac{29}{2700000}$
 $\times 5897 = -1.394$, which being applied to the number
 above, there results 5895.726; and this again being di-
 minished by its $\frac{1}{18000}$ th part, or .327, there remains
 5895.399 feet, for the correct measure of the base $\alpha\gamma$
 in the vale of Rannoch.

There is no occasion here to explain the manner of
 measuring these two bases by the twenty-foot rods, as that
 has been very circumstantially done in vol. LXV. of the
 Phil. Transf. for the year 1775, by the rev. Dr. MASKE-
 LYNE, the learned and accurate conductor of this very
 important experiment.

The following shorter lines were also measured as they
 happened to be wanted in different parts of the survey.

	Feet.	Inch.	
$\alpha'd' =$	269	4	} nearly horizontal.
$Nn =$	93	6	
$Ke =$	94	10	
$KE =$	240	10	
$ac =$	9	9	
$an =$	7	10	
$cn =$	1	11	
$ma =$	70	11	} not horizontal.
$mt =$	68	3	
$mp =$	63	4	
$pt =$	27	2	

The other measures that were taken for determining the sections will be delivered afterwards, when the results or computed altitudes have been obtained, in order to be placed opposite to their correspondent angles.

Having now obtained, to a great degree of accuracy, the measured lengths of two lines which were to serve as bases for all the future calculations, the next consideration was how to make the properest use of them. Every other line or distance, drawn or conceived to be drawn, must be calculated from them by the help of the angles observed either at their extremities, or at all the other points and stations in the survey and plan. As these two bases are situated in the low parts of the country, from whence but a very few of the other principal stations are visible, one method evidently is to compute immediately from these bases such of the great lines in the survey whose extremities are visible from them; and then from these calculated lines to compute others next to them, and so on quite around and within the whole figure. In this manner several values of each line will arise, both from the double computations by the two measured bases, and from the various sets of triangles which can be formed from the very numerous horizontal angles which were observed at the several stations. But in this mode of computation, after

great labour and pains, I had frequently the mortification to find that the several values of the same lines would differ so greatly one from another, that I was often very doubtful whether I could rely on any of them, or even on the mean among them all. These differences arose from the small errors in the observed angles, which in some degree are unavoidable; and indeed they were so small, that the sum of the angles of the several triangles which were used in the calculation seldom differed by more than a minute or two from 180° . But in a long connected chain of triangles, dependant on one another, the effects of such small errors at length become too great to be tolerated in a computation requiring much accuracy. Another method is, first to compute from both bases the length of the line KN extended along the ridge of the hill from East to West, and from it, as a secondary base, compute all the other lines in the plan. This method admits of much more accuracy than the former, supposing this secondary base to be truly assigned; because that, from the elevated and central situation of this line, all or most of the other points in the survey are visible from one or both of its extremities, by which it happens that the other lines are mostly determinable from it alone, without so close a connection with one another as in the other method of computation. By
both

both of these methods then, and by all the triangles furnished by each of them, I computed all the principal lines in the plan, and either took a mean among the several values of each, or else selected out of them such one as from various circumstances I judged it safest to rely upon, as nearest the truth. The trigonometrical computations were always accurately made, and generally repeated by logarithms, and the result of every proportion determined to two or three places of decimals. I shall here abstract the mean or corrected values of some of the principal lines or horizontal distances so computed, as well as the secondary base KN from the Eastern to the Western cairn.

The mean among a great number of ways of computation from the South base gives the horizontal distance from K to N = 4052.2, and the mean of all the results from the North base $\alpha\beta\gamma$ gives KN = 4058.9, and the mean between these two gives 4055.5 for the mean distance of K and N. And this value of KN was used in computing most of the other lines, whose mean results are as here follows.

$\alpha\gamma = 5895.4$ the Northern base in Rannoch.

$RB'' = 3011.4$ the Southern base in Glenmore.

$NK = 4055.5$ the distance of the two cairns.

RA = 5670	NR = 5545	KR = 5952	OR = 3582
AB = 1489	NE'' = 6053	KF = 8227	OB'' = 5466
BC = 4506	NA = 5941	KG = 8036	OA = 6769
CD = 775	NB = 6573	KH = 7748	OS = 3271
DF = 7388	NC = 7797	KW = 7603	OX = 4079
FG = 1166	ND = 7657	KL = 8335	OU = 6061
GH = 4068	NF = 5980	KY = 10008	OZ = 9073
HW = 2118	NG = 6370	KV = 10215	OM = 3317
WL = 1816	NH = 8195	KO = 2615	
LY = 7085	NW = 9059	KP = 3221	
YV = 3636	NL = 10405	K α = 13710	MS = 381
VT = 2645	NY = 13752	K β = 15404	Te = 1335
TZ = 4393	NS = 5795	KM' = 1817	Ze = 3719
ZU = 4132	NO = 2875	K δ' = 2528	F'D = 6430
UX = 1984	NP = 3271	Ka = 3326	F'F = 3934
XS = 2378	N α = 11876	Kb = 4409	F't' = 4098
SR = 1410	N α = 5899		t'k = 2327
	Nb = 7614		M'b = 1172
	Nb = 3381	PG = 4815	ab = 1843
	N δ' = 1585	PH = 5196	cd = 1750

From the three first lines, or bases, and the horizontal angles observed at the several stations, a very large and accurate plan of the whole survey was constructed, forming a map of four feet long by four feet broad, which was verified in every part by the measures of the computed

puted lines, both those above-written and others, and they were generally found to agree very exactly, according to the scale by which the plan was constructed. The use of this large map was to receive and admit of the distinct and accurate exhibition of the figures in their true places, expressing the number of feet in elevation or depression with respect to each observatory of every point and section of the ground whose elevation or depression might be observed. But before I proceed to the computation and construction of the points in the sections, I shall here abstract the numbers which express the relative elevation of the principal original points in the survey, being the extremes of the lines whose lengths are above abstracted. These few numbers are the results of the calculation of several hundreds of triangles conceived in a vertical position, their bases being either the horizontal lines above-written, or other lines drawn as diagonals between many distant points in the survey, according to the number of vertical angles which had been observed; and of these bases, whether real or imaginary, each generally afforded two vertical triangles, as the angles of elevation and depression were taken alternately at both ends of the lines. It is scarcely necessary to remark, that all these triangles are right-angled, the common base being one of the sides about the right angle, and the other the difference in altitude between the two

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given points or extremes of the base; and this difference in altitude is found from the application of this proportion, as radius is to the tangent of the angle of elevation or depression, so is the given base to the altitudinal difference between the two given points, exclusive of the height of the theodolite or other instrument, which was afterwards allowed for. From the resolution of all these triangles, and taking the means of the many corresponding results, were obtained the following numbers, which shew how many feet the points denoted by the letters standing against them are below the level of the point N or the Western cairn. They are all referred to this point N at the Western extremity of the ridge of the hill, because it is the most elevated point in the whole survey.

O 1184	γ 2898	H 2143	U 1613	e 2145
P 1457	A 1303	W 2024	X 1996	M 1958
K 480	B 1313	L 2006	S 1964	M' 322
R 1948	C 1384	Y 2335	a 1012	F' 2246
B'' 1920	D 1445	V 2119	b 823	t' 2815
α 2898	F 1904	T 2114	c 1364	k 2835
β 2901	G 1935	Z 1815	d 1539	δ' 172

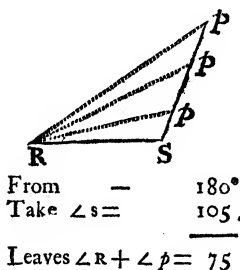
These depressions, and those of several other principal points, were first carefully computed by means of various different bases, as so many places from whence the sections were to commence.

These sections are very numerous, made in all directions from the primitive points before mentioned, and many of them extended to great distances far beyond the bounds of the plan hereunto annexed, so as to include the nearest hills and valleys of the surrounding country. They are mostly made in vertical planes in the manner described in the article of the Phil. Transf. before referred to, excepting some few of them which are level sections in planes parallel to the horizon, and some indeed irregular as being neither vertical nor horizontal. To compute the relative altitude of each point in these sections, it is evident, requires the resolution of two different triangles, *viz.* a horizontal triangle by which its place in the plan is ascertained, and a vertical triangle of which one side is the elevation or depression of the point. Of these sections there are above 70, containing near 1000 points, whose places in the plan and relative altitudes have been computed: so that the number of triangles, whose numerical resolutions have been performed in the course of this business, amounts to several thousands.

Before the abstract of the computation of the sections, I shall here put down at large the calculation of one of them, to shew the manner in which they have been computed in the readiest and easiest way that occurred to me, preserving at the same time the proper degree of accuracy.

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racy. I shall for this purpose select the third section as not containing so many poles as some of the others. This section commences at s, and is carried up the hill in a vertical plane, making an angle of 105° with the line rs. The direction of this plane is here represented by the line $sppp$ making with rs the angle $\angle rsp = 105^\circ$. The points ppp &c.



mark the places of the poles, whose angles of elevation or depreffion were taken at s with a proper instrument, and they are written in the second column of the table in this example. At R were observed the several horizontal angles, which lines supposed to be drawn from thence made with rs, and these are placed in the third column. And since in every triangle rsp , the angle s is constant, and the sum of R and p is equal to the constant quantity 75° : therefore each of the angles R, or the numbers in the third column, being subtracted from 75° , there remains the corresponding angle p: and these remainders are placed in the fourth column. Then, since the method of solution is this, as $f.p : f.R :: RS : sp = \frac{f.R}{f.p} \times RS$; and again, as radius (1) : tang. elev. :: $sp : \text{alt. of } p \text{ above } s = sp \times \text{tang. elev.} = \frac{f.R}{f.p} \times RS \times \text{tang. elev.}$. Or in logarithms

$\log. R - \log. p + \log. RS + \text{tang. elev.} = \log. \text{ of the altitude of the point.}$
 Wherefore having taken, from a table, the fines of R and p , and placed them in the fifth and sixth columns, subtract the latter from the former, and write the remainders in the next or seventh column; to these add the constant logarithm of RS , and write the sums in the eighth column; take out then the tangents of the angles in the second column, and having placed them in the ninth column, add together the adjacent numbers of the eighth and ninth columns, placing the sums in the tenth column, which being the logarithms of the altitudes or depressions of the points p , take the corresponding numbers from a table of logarithms, and write them in the eleventh or last column, for those altitudes or depressions with respect to the point s , with the height of the theodolite included, and which is afterwards allowed for, its height being generally about $4\frac{1}{2}$ or $4\frac{3}{4}$ feet. In the second column D denotes depression and E elevation; in the last column D denotes depression and A altitude.

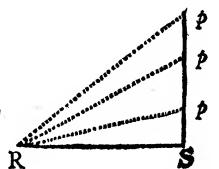
No of Poles	Angles of Dep. and Elev. at s.	Horiz. Ang. at R.	$75^{\circ} - Z's$ R, or angles p.	Sines of R.	Sines of p.	Sin. R - fin. p + rs, or sp.	Tang. of Dep. and Elev.	Sum of 8 and 9, or Log. alt. below Dep. and Alt. or above S.	Dep. and Elev.
1	3 27D	19 11	55 49	9'51666	9'91763	9'59903	2'74843	8'78022	1'52865 34D
2	2 36E	30 24	44 36	9'70418	9'84643	9'85775	3'00715	8'65715	1'66430 46A
3	4 33	38 28	36 32	9'79383	9'77473	0'01910	3'16850	8'90080	2'06030 117
4	6 12	44 10	30 50	9'84308	9'70973	0'13335	3'28275	9'03597	2'31872 208
5	7 51	47 55	27 5	9'87050	9'61828	0'25222	3'40162	9'13948	2'54110 348
6	10 38	51 22	23 38	9'89574	9'60302	0'29272	3'44212	9'27357	2'71569 520
7	12 20	53 9	21 51	9'90320	9'57075	0'33245	3'48185	9'33974	2'82159 663
8	13 46	54 43	20 17	9'91185	9'53991	0'37194	3'52134	9'38918	2'91052 814
9	15 43	56 21	18 39	9'92035	9'50486	0'41549	3'56489	9'44933	3'01422 1033
10	17 35	57 47	17 13	9'92739	9'47127	0'45612	3'60552	9'50092	3'10644 1278
11	18 0	58 58	16 2	9'93291	9'44122	0'49169	3'64109	9'51178	3'15287 1422
1	2	3	4	5	6	7	8	9	10 11

Thus

Thus then every line in the table contains the solutions of the two triangles, the one horizontal and the other vertical; used in finding the altitude of each point or pole in the section. The addition of the constant logarithm of the base RS to the logarithms in the seventh column, is most easily performed by writing it on the bottom of a little slip of paper, and so sliding it down successively over each of those numbers, and in that position adding them together, and placing the sums immediately opposite in the next column.

And in this manner were computed the relative altitudes of the points in the other vertical sections; excepting two or three cases, in which the constant angle formed by the section and the base was a right angle; and one case in which the vertical angles were not taken at the beginning of the section line, but at the other end of the base line where the horizontal angles were also observed. It may be necessary, therefore, to insert and explain an example of each of these cases, and the more so as they point out the properest means of measuring these sections so as to save most part of the labour in the computation, in which the trouble chiefly consists.

Of the case of the right angle, the first section is an instance, where also RS is the base as before, and the angle $RS\phi$ being $= 90^\circ$.



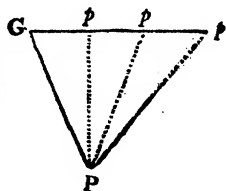
Poles.	Dep. and Elev. at s.	Horiz. Ang. at R.	Tang. of vert. $\angle$'s at s.	Tang. of $\angle$'s R.	Sum of Columns 4 and 5.	6th + rs = Log. Alt.	Depth and Alt.
1	$5^{\circ} 16\frac{1}{2}^{\circ}$ D	$10^{\circ} 0'$	8.96533	9.24632	8.21165	1.36105	23 ^D
2	$0^{\circ} 30'$ E	$31^{\circ} 35'$	7.94086	9.78874	7.72960	0.87899	$7\frac{1}{2}^{\circ}$ A
3	$4^{\circ} 15'$	$41^{\circ} 56'$	8.87106	9.95342	8.82448	1.97388	94
4	$6^{\circ} 14\frac{1}{4}'$	$49^{\circ} 25'$	9.03861	10.06722	9.10583	2.25523	180
5	$8^{\circ} 16'$	$55^{\circ} 51\frac{1}{2}'$	9.16224	10.16870	9.33094	2.48033	302
6	$10^{\circ} 13'$	$59^{\circ} 57\frac{1}{2}'$	9.25582	10.23783	9.49365	2.64305	440
7	$11^{\circ} 37'$	$62^{\circ} 56\frac{1}{2}'$	9.31297	10.29174	9.60471	2.75411	568
8	$12^{\circ} 25'$	$65^{\circ} 3\frac{3}{4}'$	9.34276	10.33257	9.67533	2.82472	668
9	$13^{\circ} 21'$	$66^{\circ} 41\frac{1}{2}'$	9.37532	10.36568	9.74100	2.89040	777
10	$14^{\circ} 10'$	$67^{\circ} 36\frac{1}{4}'$	9.40212	10.38519	9.78731	2.93671	864
11	$15^{\circ} 17'$	$68^{\circ} 42\frac{1}{2}'$	9.43657	10.40925	9.84582	2.99522	989
12	$17^{\circ} 46'$	$70^{\circ} 58'$	9.50572	10.46221	9.96793	3.11733	1310
13	$19^{\circ} 33'$	$72^{\circ} 48'$	9.55035	10.50927	10.05962	3.20902	1618
14	$20^{\circ} 6'$	$74^{\circ} 30'$	9.56342	10.55701	10.12043	3.26983	1861
1	2	3	4	5	6	7	8

In this form there are three columns less than in the former, by which it happens, that about one-third of the labour is saved. The method of solution is thus; as 1 (radius) : tang. R :: RS : sp = RS $\times$ t. R; and again, as 1 : tang. s (vertical angle) :: sp : sp $\times$ t. s = RS $\times$ t. R. $\times$ t. s. Or, in logarithms, log. RS + t. R + t. s = log. of the vertical perpendicular: and by this theorem, it is evident, the columns of this table are constructed.

But

But nearly the same saving in the great labour of computation would be made if the vertical and horizontal angles had both been taken at the end of the base farthest from the beginning of the section. And this method would also be much the easiest in making the survey on the ground, as there would then need only one observer with an instrument to measure both horizontal and vertical angles; and any person, without an instrument, could direct in a line the person who moves and places the poles, or he may even direct himself after his first pole has been placed, by means of a back object, as is commonly done in land surveying.

Of this kind there happens to have been one section taken, proceeding from G, and making with GP an angle of 85° , P being the Northern observatory, and where both the bearings and depreffions of the points *p* in the section line were observed.



$$\begin{aligned} \angle G &= \frac{180^\circ}{85} \\ \angle P + \angle p &= 95 \end{aligned}$$

	Log.	} which is a constant number from which the fines of <i>p</i> in the fifth column are to be deducted.
PG	3.68262	
f.G	9.99834	
Sum	<u>3.68096</u>	

Poles.	Vertic. Angles at P.	Horizont. Ang. at P.	$95^\circ - \angle P$ $= \angle p$	Sines of $\angle$'s at p.	$PG + s.G$ $- s.p = Pp$	Tang. of Depr.	Sum of Col. 6 and 7. = Pp.	Dep. below P.
1	8 45 ^D	9 15	85 45	9.99880	3.68216	9.18728	2.86944	740
2	8 46	16 25	78 35	9.99132	3.68964	9.18812	2.87776	755
3	9 58	27 16	67 44	9.96634	3.71462	9.24484	2.95946	911
4	8 38	30 52	64 8	9.95415	3.72681	9.18136	2.90817	809
5	7 6	34 30	60 30	9.93970	3.74126	9.09537	2.83663	686
6	5 23	37 55	57 5	9.92400	3.75696	8.97421	2.73117	538
1	2	3	4	5	6	7	8	9

Here it is evident is a saving of two of the most laborious columns in the table. This happens because that in every triangle PGp there are now constant those two parts which are used in the proportion made use of in the calculation, *viz.* PG and the angle G . For then it is, as $f.p : f.G :: PG : Pp$, or $\log. Pp = \log. PG + \angle G - f.p$; so that the sum of the logarithms of PG and sine of $\angle G$ is a constant number, from which the numbers in the fifth column are to be subtracted, to find those in the sixth column. The rest of the work is the same as in the first example.

As to the irregular sections, the computation of them differs so little in manner from that of the usual vertical sections,

sections, that an example of it is unnecessary: and the few horizontal sections need no computation, but only an allowance for the height of the theodolite.

In the following abstract of the results of the computation of the sections, the first column contains the number of the pole, the second and third the vertical and horizontal angles, and the last the difference of altitude in feet, between the foot of each pole and the point from whence the vertical angles were observed, after making the allowance for the height of the theodolite above the ground. At the end of this abstract is a plate of the figures referring to the number of the section, shewing the direction in which it was carried, with the degrees and minutes in the angle formed by it and the base line.

SECTION 1.				10	15 12½	67 36	972
Pole.	Vert. ∠'s at s.	Bearings at R.	Diff. of Alt.	11	16 24	68 45	1118
1	0 16½ D	10 0	18 D	12	17 45	70 0	1302
2	0 30 E	31 35	12 A	13	18 45	71 3	1467
3	4 15	41 56	99	14	19 35	71 57½	1625
4	6 14½	49 25	185	15	19 59	72 34	1726
5	8 16	55 51½	307	SECTION 3.			
6	10 13	59 57½	444	Pole.	Vert. ∠'s at s.	Bearings at R.	Diff. of Alt.
7	11 37	62 56½	572	1	0 27 D	19 11	29 D
8	12 25	65 3½	673	2	2 36 E	30 24	51 A
9	13 21	66 41½	782	3	4 33	38 28	122
10	14 10	67 36½	869	4	6 12	44 10	213
11	15 17	68 42½	994	5	7 51	47 55	352
12	17 46	70 58	1315	6	10 38	51 22	524
13	19 33	72 48	1623	7	12 20	53 9	668
14	20 6	74 30	1866	8	13 46	54 43	818
SECTION 2.				9	15 43	56 21	1038
Pole.	Vert. ∠'s at s.	Bearings at R.	Diff. of Alt.	10	17 35	57 47	1283
1	0 340 D	20 46	30 D	11	18 0	58 58	427
2	1 26 E	32 42	28 A	SECTION 4.			
3	4 20	42 8	103	Pole.	Vert. ∠'s at R.	Bearings at s.	Diff. of Alt.
4	6 21	49 30	192	1	7 42 D	13 8	39 D
5	9 58½	59 2	429	2	1 2 E	31 35	19 A
6	11 50½	62 30	591	3	4 4	40 24	80
7	12 52	65 2	721	4	5 36	48 36	137
8	13 22	65 51	780	5	6 55	56 56	215
9	13 36½	66 9	806				

				SECTION 6.			
	$\begin{smallmatrix} 0 & 10 \\ 9 & \end{smallmatrix}$	$\begin{smallmatrix} 62 & 36 \\ 68 & \end{smallmatrix}$	337	Pole.	Vert. $\angle$'s at x.	Bearings at s.	Diff. of Alt.
7	9 23	68 12	415				
8	10 17	70 33	495	1	$\begin{smallmatrix} 9 & 16 \\ 9 & \end{smallmatrix}$ D	$\begin{smallmatrix} 11 & 43 \\ 16 & \end{smallmatrix}$	80 D
9	11 18	74 0	623	2	4 46 D	16 51	60 D
10	13 2	76 3	785	3	1 10 E	23 50	28 A
11	13 37	76 47	848	4	4 51	30 2	137
12	14 30	77 45	946	5	6 6	33 14	196
13	15 3	78 58	1042	6	7 32	37 25	287
14	16 15	80 11	1200	7	9 30	40 24	409
15	17 24	81 13	1362	8	11 15	43 12	546
16	18 32	82 5	1528	9	12 22	45 18	656
17	19 29	82 59	1699	10	13 35	47 9	782
18	20 7	83 30	1812	11	15 19	48 54	955
				12	16 29	50 12	1093
				13	17 8	51 1	1171
				14	17 27	51 48	1248

SECTION 5.			
Pole.	Vert. $\angle$'s at s.	Bearings at x.	Diff. of Alt.
1	$\begin{smallmatrix} 3 & 46 \\ 0 & \end{smallmatrix}$ D	$\begin{smallmatrix} 15 & 0 \\ 24 & \end{smallmatrix}$	38 D
2	0 14 E	24 23	9 A
3	2 16	36 14	60
4	3 29	46 5	112
5	4 21	54 24	163
6	5 48	63 55	258
7	7 2	71 30	361
8	9 3	76 30	513
9	11 25	81 18	720
10	12 55	84 16	874
11	13 54	87 22	1017
12	14 51	90 30	1182
13	15 9	93 0	1295

SECTION 7.			
Pole.	Vert. $\angle$'s at x.	Bearings at s.	Diff. of Alt.
1	$\begin{smallmatrix} 7 & 30 \\ 9 & \end{smallmatrix}$ D	$\begin{smallmatrix} 9 & 30 \\ 15 & \end{smallmatrix}$	60 D
2	1 49	15 52	24
3	2 5 E	20 49	52 A
4	4 30	24 48	135
5	5 41	28 23	208
6	7 13	30 27	297
7	8 14	32 29	380
8	9 24	33 41	465
9	10 26	35 0	558

10	11 41	36 21	678
11	12 36	37 15	773
12	13 31	38 17	885
13	13 46	39 30	973
14	13 56	40 17	1036

between A and N is only 1303 feet. This diff. of 13 feet seems to be caused by the last bearing being about 7' too great, for in other places this angle is only $58^{\circ} 51'$. And indeed many other angles taken at the same time with the above seem to be much wrong, as they greatly differ from corresponding ones taken at other times.

Such differences among corresponding angles I often met with in the measures contained in the books of the survey, and it required much care to detect them, and trouble to reconcile them.

SECTION 8.

Pole.	Vert. $\angle$'s at A.	Bearings at B.	Diff. of Alt.
1	17 56½ D	37 12	516 D
2	10 12½	41 50	359
3	5 41	44 48	229
4	2 25	47 2	107
5	0 29	49 6	20
6	1 12 E	51 0	74 A
7	3 0	52 35½	198
8	5 0	54 44½	378
9	6 18	55 56½	520
10	7 2	56 47½	620
11	8 52	57 24	821
12	10 10	57 55½	986
13	11 24	58 28	1160
14	12 20½	58 58	1316

N.B. The place of this last pole would seem to be the same as N the Western cairn, as the section was directed through it. But then the last number 1316 is too great; as, from all the other measures, the diff. in alt.

SECTION 9.

Pole.	Bearings at A.	Bearings at B.
1	132 21½	36 45½
2	130 27	37 34½
3	127 26	39 8
4	124 25½	40 18
5	119 6½	43 19
6	111 37	48 7
7	103 45½	52 58
8	95 25	58 50
9	85 55	67 54
10	78 2½	75 58½
11	73 14	82 30
12	71 41½	86 21
13	70 38½	89 33
14	69 39	92 23

This is a horizontal or level section through A, and therefore each point is $4\frac{1}{2}$ feet (the height of the theodolite) above that point.

SECTION IO.				13	9 44	76 12½	1168
Pole.	Vert. ∠'s at A.	Bearings at B.	Diff. of Alt.	14	9 53	76 34	1258
				15	10 38	76 55½	1443
1	19 41 D	51 12	531 D	SECTION I2.			
2	13 52	57 22½	426	Pole.	Bearings at D.	Bearings at c.	Each of these poles is five feet above D, the section being horizontal and taken at that point.
3	8 7½	62 56	295	1	68 14½	102 6	
4	2 30	68 47	106	2	71 16	98 23	
5	0 59	73 55½	47	3	73 55	94 56	
6	0 22½ E	76 52	27 A	4	79 0	89 33	
7	0 59	79 14½	70	5	85 28	82 49	
8	1 39	81 7	124	6	91 19	76 48	
9	2 25	82 42	194	7	97 5	71 8	
10	3 5	84 12	266	8	102 21	66 20	
11	3 36	85 41½	338	9	108 18	60 36	
12	3 48	86 27	373	10	113 33	56 15	
SECTION II.				11	117 12	53 17	
Pole.	Vert. ∠'s at D.	Bearings at c.	Diff. of Alt.	12	119 16	52 4	
1	12 56 D	48 18	231 D	13	119 53	52 1	
2	9 50	62 4	334	SECTION I3.			
3	5 15	67 9	242	Pole.	Vert. ∠'s at G.	Bearings at F.	Diff. of Alt.
4	2 25	69 51½	135	1	3 7 D	75 2	92 D
5	0 59 E	71 20	70 A	2	0 12	79 46	2
6	3 0	72 13	220	3	4 1 E	91 55	221 A
7	4 31	73 9	363	4	6 8	95 0	385
8	6 7	73 53	533	5	7 49	96 34	530
9	6 59	74 27	652	6	9 50	99 27	789
10	7 53	74 52	777	7	10 52	100 10	915
11	8 49	75 9½	904				
12	9 27	75 42	1048				

8	11 52	100 44	1040	5	6 27	45 0	465
9	12 52	101 15	1172	6	8 10	47 32	642
10	13 56	101 49	1327	7	9 10	49 47	781
11	14 49	102 20	1472	8	10 39	51 37	970
12	16 10	103 3	1710	9	12 0	53 36	1177
13	16 45	103 27	1837	10	13 24	55 38	1422
14	16 55	104 20	2013	11	14 0	57 18	1584
SECTION 14.							
12	14 18	58 37	1704	13	14 21	58 58	1734

Pole.	Vert. $\angle$'s at G.	Bearings at F.	Diff. of Alt.
1	2 3 E	97 5	73 A
2	2 35	99 14	95
3	3 52	109 39	182
4	5 58	117 53	384
5	6 25	119 8	440
6	8 31	120 34	633
7	10 14	121 51	827
8	11 29	122 41	985
9	12 30	123 34	1149
10	13 4	124 15	1272
11	13 14	124 41	1339

SECTION 15.			
Pole.	Vert. $\angle$'s at G.	Bearings at H.	Diff. of Alt.
1	9 46 D	14 14	172 D
2	3 32	23 9	102
3	0 45	31 7	27
4	1 43 E	36 15	94 A

SECTION 16.			
Pole.	Vert. $\angle$'s at H.	Bearings at G.	Diff. of Alt.
1	7 22 D	21 37	189 D
2	6 11	34 32	248
3	4 1	43 23	201
4	0 9	48 20	4
5	1 43 E	54 38	119 A
6	3 35	60 8	275
7	5 14	65 8	446
8	6 49	70 13	653
9	8 1	73 19	828
10	8 41	76 39	977
11	9 18	78 38	1104
12	10 0	80 22	1246
13	10 44	82 0	1403
14	11 31	83 24	1572
15	13 8	84 43	1874

There seems to be some general error in this section, as the depreffions and altitudes are utterly incompatible with those of all the other neighbouring points in the plan.

SECTION 17.							
Pole.	Vert. $\angle$'s at H.	Bearings at G.	Diff. of Alt.		$^{\circ}$ $'$	$^{\circ}$ $'$	
1	12 15 D	16 13	242 D	8	6 55	78 7	531
2	7 11	25 6	216	9	8 33	81 13	737
3	4 19	33 3	171	10	9 28	88 4	1108
4	1 49	38 20	82	11	9 38	89 10	1201
5	1 59 E	45 24	119 A	12	9 53	89 58	1290
6	3 38	50 3	243	13	10 24	90 33	1407
7	5 10	53 33	377	14	11 2	91 10	1552
8	7 10	57 3	572	SECTION 19.			
9	8 17	60 46	731	Pole.	Vert. $\angle$'s at w.	Bearings at L.	Diff. of Alt.
10	8 45	62 58	820	1	18 8 D	14 51	168 D
11	9 53	66 56	1038	2	16 28	31 38	388
12	10 24	69 0	1161	3	11 53	39 42	394
13	10 45	70 35	1260	4	7 47	42 48	292
14	11 31	72 15	1424	5	4 11	46 12	180
15	12 10	73 55	1590	6	2 10	48 6	100
16	12 31	75 7	1703	7	0 31	50 16	23
SECTION 18.				8	2 24 E	52 45	148 A
Pole.	Vert. $\angle$'s at H.	Bearings at w.	Diff. of Alt.	9	5 55	54 48	402
1	13 35 D	21 40	186 D	10	8 48	56 30	657
2	9 17 1/2	43 28	263	SECTION 20.			
3	5 21	55 3	203	Pole.	Vert. $\angle$'s at w.	Bearings at L.	Diff. of Alt.
4	2 4	63 8	95	1	22 45 D	7 37	98 D
5	1 23 E	69 50	84 A	2	19 0	16 20	179
6	3 37	73 24	238	3	18 41	31 8	367
7	5 25	76 0	386	4	14 20	41 54	412
				5	9 41	48 4	341

6	3 38	54 6	155	9	14 40	43 34	874
7	1 15	56 50	57	10	14 45 top of E. cairn.	43 35	874
8	0 55 E	60 7	55 A				
9	4 4	62 58	257				
10	6 45	66 2	487				
SECTION 21.				SECTION 23.			
Pole.	Vert. $\angle$'s at L.	Bearings at w.	Diff. of Alt.	Pole.	Vert. $\angle$'s at a.	Bearings at b.	Diff. of Alt.
1	23 59 D	21 18	295 D	1	15 2 D	4 33	34 D
2	19 25	33 12	377	2	0 51	15 15	4
3	14 37	48 26	448	3	0 10	35 37	0
4	10 5	56 7	383	4	3 7 E	41 53	97 A
5	5 9	62 51	237	5	6 43	42 34	209 -
6	1 2	69 24	56	6	7 36	43 30	244
7	2 2 E	71 36	133 A	7	9 50	45 48	342
8	4 22	73 2	297	8	11 35	59 5	664
9	6 6	75 22	455				
SECTION 22.				SECTION 24.			
Pole.	Vert. $\angle$'s at a.	Bearings at b.	Diff. of Alt.	Pole.	Vert. $\angle$'s at b.	Bearings at a.	Diff. of Alt.
1	7 16 D	5 2	20 D	1	3 27 D	10 29	29 D
2	2 49 E	10 20	25 A	2	0 32 E	21 9	12 A
3	3 33	15 13	44	3	2 58	32 57	64
4	4 18	29 34	120	4	4 40	58 53	133
5	7 0	32 15	222	5	5 14	76 22	173
6	9 1	35 3	331	6	6 24	121 5	340
7	10 26	38 52	471	7	7 50	125 12	448
8	12 19	41 46	657	8	8 27	130 23	546
				9	8 55	134 57	664
				10	9 6	136 15	712

SECTION 25.							
Pole.	Vert. $\angle$'s at <i>b</i> .	Bearings at <i>a</i> .	Diff. of Alt.				
1	15 39 D	8 14	74 D	6	2 20	42 15	69
2	1 47 E	52 24	54 A	7	4 50	37 30	122
3	3 9	63 13	115	8	18 52	6 12	65
4	5 51	70 37	248	SECTION 28.			
5	9 11	81 35	505	Pole.	Vert. $\angle$'s at <i>T</i> .	Bearings at <i>V</i> .	Diff. of Alt.
6	11 21	85 10	691	1	10 23 D	16 36	142 D
7	12 44	87 45	802	2	9 26	23 56	195
SECTION 26.				3	5 23	28 45	136
Pole.	Vert. $\angle$'s at <i>b</i> .	Bearings at <i>a</i> .	Diff. of Alt.	4	4 14	32 24	120
1	17 53 D	6 35	64 D	5	2 42	37 46	96
2	0 26	48 5	9	6	1 34	41 20	62
3	2 1 A	60 8	93 A	7	0 35 $\frac{1}{2}$	45 24	24
4	3 4	62 45	151	8	1 30 E	48 52	89 A
5	4 40	66 15	256	9	3 30	51 4	220
6	6 20	68 29	374	10	4 20	54 22	307
7	8 10	71 57	550	11	4 49	56 11	367
SECTION 27.				12	5 15	58 41	443
Pole.	Vert. $\angle$'s at <i>b</i> .	Bearings at <i>a</i> .	Diff. of Alt.	13	6 0	60 4	537
1	4 30 E	59 25	280 A	14	6 47	62 4	664
2	3 33	57 12	204	15	7 24	63 33	778
3	1 47	54 11	92	16	8 16	64 46	924
4	0 10 D	50 15	3 D	17	8 46	65 44	1030
5	1 22	46 54	47	SECTION 29.			
				Pole.	Vert. $\angle$'s at <i>T</i> .	Bearings at <i>V</i> .	Diff. of Alt.
				1	8 49 D	26 9	182 D
				2	6 51	30 7	163

3	0 27	38 45	140	13	8 3	29 13	143
4	2 55	44 0	106	14	10 2	21 54	170
5	1 30	49 34	62				
6	0 10	52 56	3				
7	0 42 E	55 55	42 A				
8	2 12	58 28	132				
9	3 3	60 58	195				
10	3 36	63 32	247				
11	4 4	66 16	303				
12	4 8	68 28	331				
13	4 27	70 9	377				
14	5 0	71 54	450				
15	5 7	74 23	506				

SECTION 30.							
Pole.	Vert. $\angle$'s at r.	Bearings at v.	Diff. of Alt.	Pole.	Vert. $\angle$'s at r.	Bearings at v.	Diff. of Alt.
1	0 26 E	81 0	353 A	1	11 18 D	10 40	132 D
2	3 10	79 53	313	2	5 29	15 43	93
3	2 55	77 7	263	3	2 56	19 13	60
4	2 46	75 2	233	4	1 42	22 49	40
5	2 35	72 45	204	5	0 42	28 23	20
6	2 26	70 16	179	6	0 16	31 43	5
7	1 20	66 28	91	7	0 30 E	36 10	28 A
8	0 19	62 32	23	8	1 33	40 41	90
9	0 49 D	57 58	37 D	9	2 13	45 0	146
10	2 33	51 54	107	10	2 41	47 18	190
11	4 23	43 37	151	11	2 54	50 47	231
12	6 21	35 48	176	12	3 13	53 12	278
				13	3 39	56 0	349
				14	4 53	58 51	519
				15	5 42	60 38	650
				16	6 0	62 8	728
				17	6 27	63 23	825
				18	6 41	64 21	892
				19	6 55	65 50	988

SECTION 32.							
Pole.	Vert. $\angle$'s at v.	Bearings at v.	Diff. of Alt.				
1	$0^{\circ} 7' E$	$51^{\circ} 46'$	812 A	9	$0^{\circ} 28' E$	$65^{\circ} 38'$	38 A
2	6 55	50 50	751	10	1 40	71 15	132
3	6 7	49 45	631	11	2 22	77 3	197
4	5 0	46 51	452	12	2 25	87 12	226
5	4 25	45 30	377	13	2 32	93 33	257
6	3 29	43 37	275	SECTION 34.			
7	3 5	40 32	214	Pole.	Vert. $\angle$'s at T.	Bearings at z.	Diff. of Alt.
8	2 17	38 26	146	1	$0^{\circ} 43' E$	$84^{\circ} 3'$	471 A
9	1 21	35 57	80	2	4 8	79 45	386
10	0 52	33 0	47	3	3 31	76 0	311
11	0 3	29 47	7	4	2 45	71 30	229
12	1 3 D	26 24	33 D	5	2 0	66 22	156
13	2 3	23 40	60	6	1 8	60 49	84
14	3 21	18 30	73	7	0 19	53 20	25
15	7 54	13 4	121	8	0 55 D	46 47	46 D
SECTION 33.				9	1 29	40 15	69
Pole.	Vert. $\angle$'s at T.	Bearings at z.	Diff. of Alt.	10	1 59	34 52	83
1	$13^{\circ} 6' D$	$4^{\circ} 6'$	108 D	11	3 41	26 15	127
2	9 15	8 4	139	12	5 27	21 55	165
3	7 32	14 53	187	13	6 0	17 16	150
4	5 10	21 28	167	14	9 5	9 56	143
5	3 55	26 54	148	15	14 6	4 32	109
6	2 36	36 19	119	SECTION 35.			
7	1 30	45 32	78	Pole.	Vert. $\angle$'s at T.	Bearings at z.	Diff. of Alt.
8	0 40	56 5	37	1	$15^{\circ} 16' D$	$1^{\circ} 58'$	86 D
				2	9 36	5 3	126

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3	8 21	8 5	162
4	7 34	12 54	208
5	5 38	17 50	191
6	4 21	22 48	170
7	3 12	32 56	153
8	2 31	41 54	135
9	1 42	54 12	103
10	1 10	64 26	76
11	0 52	75 48	61
12	0 30	86 54	36
13	0 14	94 17	15

SECTION 36.

Pole.	Vert. $\angle$'s at u.	Bearings at z.	Diff. of Alt.
1	0 17 42 D	0 2 45	60 D
2	15 55	22 26	465
3	12 39	25 13	404
4	8 51	29 37	333
5	7 23	33 36	318
6	3 51	40 33	205
7	1 22	44 45	80
8	1 3 E	48 26	77 A
9	2 54	50 52	218
10	4 22	52 45	347
11	5 37	54 40	471
12	6 30	55 30	559

SECTION 37.

Pole.	Vert. $\angle$'s at u.	Bearings at z.	Diff. of Alt.
1	0 3 53 E	0 63 50	345 A
2	3 27	60 54	286
3	1 55	57 45	150
4	0 16	54 3	23
5	1 14 D	50 40	74 D
6	2 53	47 28	166
7	4 45	43 0	247
8	6 25	38 49	300
9	8 40	33 14	348
10	12 6	28 25	420
11	16 36	23 43	419

SECTION 38.

Pole.	Vert. $\angle$'s at u.	Bearings at z.	Diff. of Alt.
1	0 15 9 D	0 20 37	405 D
2	11 30	31 39	438
3	9 23	36 7	398
4	6 46	40 30	314
5	5 27	46 10	283
6	3 32	52 5	204
7	2 49	55 9	171
8	1 49	61 25	122

SECTION 39.							
Pole.	Vert. $\angle$'s at u.	Bearings at z.	Diff. of Alt.				
1	0 28 D	69 7	312 D	7	2 8	56 26	269
2	5 35	59 8	345	8	1 21	58 30	178
3	6 30	52 3	366	9	0 57	59 53	129
4	8 9	45 25	418	10	0 43	62 21	103
5	10 32	38 30	485	11	0 40	64 35	102
6	12 47	34 3	544	12	0 5	68 41	10
7	13 47	29 27	531	13	0 10 E	69 43	36 A
				14	0 32	71 20	109
SECTION 40.				SECTION 42.			
Pole.	Vert. $\angle$'s at P.	Bearings at P.	Diff. of Alt.	Pole.	Vert. $\angle$'s at M.	Bearings at s.	Diff. of Alt.
1	8 45 D	9 15	735 D	1	0 17 D	73 48	0
2	8 46	16 25	750	2	3 25 E	80 57	72 A
3	9 58	27 16	906	3	4 52	85 4	126
4	8 38	30 52	804	4	6 52	88 53	232
5	7 6	34 30	681	5	8 33	90 25	332
6	5 23	37 55	533	6	10 2	91 34	439
				7	11 44	93 0	610
SECTION 41.				8	13 7	94 0	787
Pole.	Vert. $\angle$'s at o.	Bearings at A.	Diff. of Alt.	9	15 10	94 34	1002
1	10 41 D	23 33	510 D	10	17 21	95 13	1292
2	11 31	32 36	769	11	18 38	95 37	1506
3	8 52	37 23	682	12	19 36	96 2	1736
4	5 38	44 24	524	13	19 38	96 0	1727
5	4 52	46 39	480				
6	4 6	51 8	455	SECTION 43.			
				Pole.	Vert. $\angle$'s at o.	Bearings at A.	Diff. of Alt.
				1	12 3 D	31 52	759 D
				2	10 54	36 42	776

3	8 58	41 8	705	4	0 8 E	42 19	16 A
4	7 2	46 9	614	5	4 38	49 48	517
5	5 35	49 34	521	6	5 12	50 45	598
6	3 33	58 6	389	7	5 58	52 13	721
7	2 52	62 38	341	8	6 37	54 58	878
8	2 14	67 20	290	9	7 18	56 7	1008
9	2 12	85 34	421	10	7 22	56 39	1037

SECTION 44.				SECTION 46.			
Pole.	Vert. $\angle$'s at R.	Bearings at s.	Diff. of Alt.	Pole.	Vert. $\angle$'s at D.	Bearings at c.	Diff. of Alt.
1	3 41 D	6 37	32 D	1	8 50 D	96 20	201 D
2	1 18	10 10	21	2	7 24	111 4	260
3	0 51 E	11 21	25 A	3	6 36	114 46	272
4	0 52	12 20	30	4	6 9	116 22	275
5	1 4	16 25	70	5	5 24	119 44	298
6	2 3	16 52	143	6	5 4	121 17	315
7	3 6	17 27	243	7	4 16	122 24	292
8	4 38	18 0	413	8	3 27	123 13	255
9	5 14	18 6	478	9	2 23	124 17	197
10	5 41	18 11	530	10	1 17	124 55	113
11	6 28	18 27	647	11	0 44 E	125 50	82 A
				12	1 17	126 8	146

SECTION 45.				SECTION 47.			
Pole.	Vert. $\angle$'s at A.	Bearings at R.	Diff. of Alt.	Pole.	Vert. $\angle$'s at T.	Bearings at v.	Diff. of Alt.
1	2 15 D	30 14	122 D	1	10 42 D	17 45	152 D
2	2 0	31 57	116	2	6 6	28 48	145
3	1 22	38 35	100	3	3 19	38 30	111

4	2 4	43 5	79	15	8 52	80 32	1435
5	0 44	47 48	30	16	9 6	82 53	1641
6	0 30 E	49 46	30 A	SECTION 49.			
7	2 28	52 45	143	Pole.	Vert. $\angle$'s at r.	Bearings at e.	Diff. of Alt.
8	3 24	54 21	205	1	11 56 D	16 58	83 D
9	4 39	59 0	335	2	9 12	35 25	120
10	4 54	62 16	387	3	8 14	64 20	190
11	5 57	64 50	517	4	8 12	80 0	250
12	6 16	65 38	562	5	5 55	92 52	234
13	6 50	67 39	666	6	4 14	98 36	193
14	7 11	69 40	764	7	1 58	103 12	101
SECTION 48.				8	0 34 E	106 12	39
Pole.	Vert. $\angle$'s at v.	Bearings at r.	Diff. of Alt.	9	1 20	108 26	92
1	9 0 D	17 3	164 D	10	2 15	109 6	157
2	7 0	30 56	236	11	3 7	110 35	230
3	3 22	44 36	170	12	3 49½	111 48	300
4	1 16	47 23	66	SECTION 50.			
5	0 5	49 52	0	Pole.	Vert. $\angle$'s at L.	Bearings at w.	Diff. of Alt.
6	2 46 E	58 6	214 A	1	17 46 D	10 52	226 D
7	4 57	64 34	457	2	17 2	13 33	296
8	5 53 .	67 2	585	3	14 51	16 2	333
9	6 21	70 8	696	4	10 47	20 13	363
10	6 30	72 34	772	5	9 26	23 0	418
11	6 34	75 13	857	6	8 54	24 37	466
12	7 41	77 32	1097	7	8 7	26 16	508
13	8 39	79 6	1316				
14	8 46	79 34	1361				

8	7 8	27 23	507	4	7 6	108 57	728
9	6 17	28 27	507	5	8 4	111 38	863
10	4 55	29 28	451	6	8 37	112 58	943
11	4 30	30 12	490	7	9 19	115 56	1075
12	3 51	30 55	433	8	10 8	118 30	1230
13	3 2	31 15	357	9	10 57	120 40	1393
14	2 35	31 29	315	10	11 30	122 30	1528
SECTION 51.							
Pole.	Vert. $\angle$'s at F.	Bearings at F'.	Diff. of Alt.				
1	14 37 D	3 57	73 D	11	12 22	124 50	1748
2	2 56	44 8	140	12	13 40	127 12	2075
3	1 43	48 47	90	13	14 3	128 13	2205
4	1 3	55 5	61	14	14 7	128 32	2239
5	1 0	60 58	65	SECTION 53.			
6	0 41	67 26	50	Pole.	Vert. $\angle$'s at F'.	Bearings at F.	Diff. of Alt.
7	0 23	74 28	31	1	3 11 E	64 3	208 A
8	0 25 E	82 0	52 A	2	5 9	71 24	366
9	1 53	83 46	225	3	5 58	75 50	448
10	3 17	87 0	423	4	6 14	84 12	522
11	3 32	87 28	461	5	8 51	91 23	821
SECTION 52.							
Pole.	Vert. $\angle$'s at F'.	Bearings at F.	Diff. of Alt.	6	9 0	97 47	924
1	2 48 E	68 41	187 A	7	10 7	99 40	1074
2	3 50	83 0	282	8	11 2	102 33	1236
3	6 45	102 0	630	9	11 45	104 44	1375
				10	12 6	106 28	1469
				11	12 43	108 24	1612
				12	12 55	109 18	1673

SECTION 54.				SECTION 56.			
Pole.	Vert. $\angle$'s at t'.	Bearings at F'.	Diff. of Alt.	Pole.	Vert. $\angle$'s at t'.	Bearings at F'.	Diff. of Alt.
1	0 22 E	14 7	12 A	1	3 11 E	9 4	41 A
2	1 27	20 5	44	2	4 27	16 21	97
3	5 13	39 2	241	3	7 15	21 33	205
4	5 46	45 18	299	4	9 12	27 0	331
5	6 9	55 40	379	5	10 30	29 15	414
6	6 38	62 0	451	6	11 18	31 14	480
7	7 11	70 8	553	7	11 26	35 39	569
8	7 30	78 56	662	8	12 21	38 50	686
9	7 48	84 30	755	9	12 42	41 44	777
10	7 50	94 8	908	10	13 18	44 21	887
11	7 54	98 26	1006	11	13 32	46 36	971
12	7 58	100 58	1077				

SECTION 55.				SECTION 57.			
Pole.	Vert. $\angle$'s at t'.	Bearings at F'.	Diff. of Alt.	Pole.	Vert. $\angle$'s at k.	Bearings at t'.	Diff. of Alt.
1	2 0 E	11 46	35 A	1	7 51 E	29 43	176 A
2	2 3	22 45	62	2	11 24	34 13	299
3	4 3	25 32	130	3	13 22	37 54	397
4	7 32	34 49	324	4	14 52	42 20	509
5	9 15	42 55	497	5	16 40	47 44	675
6	9 52	48 8	604	6	16 46	50 15	833
7	10 30	53 40	736				
8	10 33	62 34	917				
9	11 7	66 12	1060				
10	11 57	69 15	1235				
11	11 52	73 19	1370				

SECTION 58.			
Pole.	Vert. $\angle$'s at k.	Bearings at t'.	Diff. of Alt.
1	0 3 E	5 28	5 A
2	6 10	8 5	44

3	10 3	13 54	121	6	21 20	83 32	1521
4	10 20	29 37	313	7	20 14	85 50	1656
5	14 20	32 18	497	8	19 12	87 30	1758
6	15 0	33 18	547	9	18 46	89 17	1978
SECTION 59.							
Pole.	Vert. $\angle$'s at M.	Bearings at h.	Diff. of Alt.	10	17 30	90 20	2016
1	28 16 D	Not teen.		11	16 8	91 5	1990
2	25 20	66 56	910 D	12	14 42	92 27	2093
3	22 23	74 50	1056	13	13 35	92 54	2035
4	22 12	76 20	1114	14	12 46	93 14	1991
5	21 40	79 10	1232	15	11 30	93 45	1915
				16	10 30	94 7	1845

The following are the irregular sections. In the first column is the number of poles; in the second the vertical angles; in the third and fourth the two bearings or horizontal angles at each end of the base; and in the fifth the computed result, being the difference of altitude between the foot of each pole and the point mentioned in the second column where the vertical angles were taken.

SECTION 60.					SECTION 61.				
Pole.	Vert. $\angle$'s at H.	Bear. at H.	Bear. at G.	Diff. of Alt.	Pole.	Vert. $\angle$'s at H.	Bear. at H.	Bear. at G.	Diff. of Alt.
1	8 30 E	70 55	66 39	832 A	1	9 31 E	68 51	71 0	1004 A
2	8 30	66 46	70 33	850	2	9 31	64 19	77 50	1091
3	8 30	62 56	75 9	884	3	9 31	60 19	80 38	1072
4	8 30	58 20	79 20	892	4	9 31	55 40	84 34	1066
5	8 30	54 27	82 42	891					

SECTION 62.					SECTION 65.				
Pole.	Vert. $\angle$'s at H.	Bear. at H.	Bear. at G.	Diff. of Alt.	Pole.	Vert. $\angle$'s at b.	Bear. at b.	Bear. at a.	Diff. of Alt.
1	0 0 E	72 45	72 40	1534A	1	2 1 D	47 47	19 56	20D
2	11 0	69 57	76 21	1389	2	1 15 E	32 1	26 45	26A
3	11 0	62 40	82 29	1376	3	2 59	27 0	32 56	65
4	11 0	59 7	84 22	1327	4	4 35	23 40	45 18	118
SECTION 63.					5	5 0	19 2	63 47	150
Pole.	Vert. $\angle$'s at H.	Bear. at H.	Bear. at G.	Diff. of Alt.	6	5 39	15 26	98 12	202
1	12 15 E	74 9	74 0	1613A	7	6 1	13 3	121 47	238
2	12 15	70 14	78 7	1652	8	5 44	9 29	143 12	247
3	12 15	67 32	80 52	1669	SECTION 66.				
4	12 15	64 2	84 1	1664	Pole.	Vert. $\angle$'s at d.	Bear. at d.	Bear. at c.	Diff. of Alt.
SECTION 64.					1	14 25 D	162 37	11 30	870D
Pole.	Vert. $\angle$'s at w.	Bear. at w.	Bear. at L.	Diff. of Alt.	2	15 55	158 15	13 36	823
1	5 57 E	74 37	81 12	382A	3	16 57	152 52	16 35	827
2	6 57	78 12	76 36	333	4	18 12	149 43	18 0	831
3	7 11	81 12	73 19	329	5	20 57	147 3	18 41	866
4	6 19	86 33	68 34	394	6	23 5	136 18	23 37	865
5	7 5	92 16	64 4	334	7	23 40	129 0	27 23	876
6	7 48	96 43	60 49	271	8	23 3	122 16	31 32	877
7	8 40	100 10	58 40	184	9	23 46	112 59	36 25	894
8	8 52	102 42	56 41	167	10	23 23	104 22	41 34	891
9	8 41	106 0	54 23	168	11	23 35	91 13	48 53	892
10	8 30	108 53	52 16	176	12	21 21	82 49	57 38	902
					13	20 11	77 33	63 0	897
					14	18 47	69 33	71 24	890
					15	17 31	65 26	77 38	893

16	16 1	61 51	83 57	884	22	10 0	32 45	65 51	841
17	14 33	59 1	89 55	875	23	9 22	29 27	71 0	819
18	12 48	56 49	96 15	868	SECTION 68.				
19	11 38	54 45	100 45	850	Pole	Vert. $\angle$'s at P.	Bear. at P.	Bear. at G.	Diff. of Alt.
SECTION 67.					1	6 23D	21 51	0	
Pole.	Vert. $\angle$'s at P.	Bear. at P.	Bear. at H.	Diff. of Alt.	2	6 20	22 44	139 42	1140D
1	5 42D	47 54	61 12	476D	3	6 20	24 2	136 17	1092
2	7 42	54 10	53 56	592	4	6 14	25 20	132 35	1025
3	7 13	55 23	52 22	542	5	5 54	27 20	127 52	932
4	7 12	58 40	50 36	532	6	5 41	29 23	121 57	843
5	8 1	63 52	46 43	564	7	5 21	32 27	115 58	769
6	6 47	68 29	44 48	469	8	5 29	35 59	107 50	740
7	5 27	71 25	43 18	369	9	5 45	41 31	87 8	615
8	5 31	79 14	40 16	368	10	5 30	45 10	82 44	578
9	4 15	81 17	39 11	278	11	5 42	49 50	77 53	589
10	3 51	87 37	36 34	247	12	5 56	53 57	74 41	613
11	3 48	90 58	35 16	242	13	6 3	58 6	70 2	605
12	1 5	96 47	32 56	64	14	5 4	64 0	66 29	510
13	1 27	96 53	30 46	80	15	5 8	67 27	65 5	527
14	3 42	91 16	32 0	208	16	4 11	73 46	65 27	486
15	6 0	83 54	34 35	348	17	4 20	76 52	64 11	518
16	7 21	69 39	38 26	435	18	4 21	80 26	62 58	542
17	9 5	64 1	41 18	564	SECTION 69.				
18	9 34	55 56	44 56	625	Pole.	Vert. $\angle$'s at H.	Bear. at H.	Bear. at w.	Diff. of Alt.
19	10 1	48 45	49 59	706	1	12 47E	116 58	51 34	1809A
20	9 24	41 45	54 41	703	2	14 36	107 42	59 25	2135
21	9 52	37 19	59 16	777					

3	13 56	101 41	65 9	2023	8	12 3	78 24	85 45	1656
4	13 43	96 35	69 7	1960	9	11 47	76 54	87 30	1646
5	13 9	92 53	72 30	1875	10	11 26	74 56	89 25	1592
6	12 38	88 41	76 6	1760	11	10 6	68 32	96 23	1452
7	12 28	82 53	81 18	1703					

The three following sections were taken in a manner different from all the rest. They were made by measuring in a straight sloping line (or nearly straight) from certain points towards K and N, and at the beginning of the line taking the angle of elevation or depression of several places or points in it, whose distance from the beginning were measured. In these cases each distance is the hypotenuse of a right-angled triangle, and the manner of operation is this, as radius is to the hypotenuse or measured slope distance, so is the sine of the elevation or depression to the difference of altitude, and so is the cosine of the same vertical angle to the horizontal distance.

SECTION 70, from M' to K.									
Pole.	Slope Dist.	Vert. $\angle$'s at M'.	Horiz. Dist.	Diff. of Alt.					
		$0^{\circ} 50\frac{1}{2}'$			4	1257	$5^{\circ} 28\frac{1}{2}'$	1257	116
					5	1455	$5^{\circ} 25\frac{1}{2}'$	1449	134
1	463		459	60	6	1824	$5^{\circ} 4'$	1817	157
2	794	$6^{\circ} 54\frac{1}{2}'$	788	92	Ends at K.				
3	992	$6^{\circ} 50'$	985	114					

SECTION 71, from δ' to K.					SECTION 72, from δ' to N.				
Pole.	Slope Diff.	Vert. $\angle$'s at δ .	Horiz. Diff.	Diff. of Alt.	Pole.	Slope Diff.	Vert. $\angle$'s at δ .	Horiz. Diff.	Diff. of Alt.
1	727	$13^{\circ} 39\frac{1}{2}'$	706	167	1	528	$3^{\circ} 10'$	527	38
2	1455	$9^{\circ} 44\frac{1}{2}'$	1434	242	2	1188	$4^{\circ} 10'$	1185	95
3	1720	$9^{\circ} 4\frac{1}{2}'$	1699	267	3	1594	$6^{\circ} 8'$	1585	179
4	1984	$7^{\circ} 52'$	1965	267	Ends at N.				
5	2547 Ends at K.	$7^{\circ} 3'$	2528	308					

Besides these sections there were many more single points, whose places and relative altitudes were observed and computed, but it is not necessary to abstract them all here.

The following plate (Tab. VIII.) has 72 figures answering to these 72 sections, each to each, according to the numbers. In these figures, the line having the letters p, p, p, &c. annexed is the section line, the letters p, p, &c. denoting the poles; the other line, forming the angle with the section, is the base line; and between them are the degrees and minutes contained in the angle formed by them; at the angular point was observed the elevation or depression of each point p, and the bearings or horizontal angles were observed at the other end of the base, from whence faint lines are drawn to some of the points p forming with the base line those horizontal angles.

angles. The base and section lines in each figure are also drawn nearly in the same direction as they are in the plan or on the ground, supposing the top of the paper to be the North, towards which a person looks when viewing the ground from the South.

Having finished the computation of the relative altitudes of all the points, the next consideration is how they are to be applied in determining the attraction of the hill. In whatever manner this last mentioned operation may be performed, it is evident, that all the points and sections with their altitudes must be entered in the plan. Wherefore, having accurately constructed a large plan of the ground, as before mentioned, containing all the principal lines or bases, at the extremities of which either vertical or horizontal angles were taken, from them I then determined in this plan the places of all the other points in the sections, whether vertical, horizontal, or irregular. These places or points were determined by drawing lines from each extremity of the base so as to form with it angles equal to those which were observed on the ground for each corresponding pole; the intersections of these lines are the places of the poles, which having marked with a fine dot or point of ink, and written close to each point the proper number expressing its relative altitude, and cleaned the paper by rubbing out the lines forming the

angles by which the points were determined, there remained only the points with the figures expressing their altitudes distinctly exhibited in the plan (see tab. IX.)

It remains now to apply all the foregoing calculations and constructions to the determination of the effect of the attraction in the direction of the meridian. And here it soon occurred, that the best method was to divide the plan into a great number of small parts, which may be considered as the bases of as many vertical columns or pillars of matter into which the hill and the adjacent ground may be supposed to be divided by vertical planes, forming an imaginary group of vertical columns, something like a set of basaltine pillars, or like the cells in a piece of honey-comb; then to compute the attraction of each pillar separately in the direction of the meridian; and lastly, to take the sum of all these computed effects for the whole attraction of the matter in the hill, &c. Now the attraction of any one of these pillars on a body in a given place may be easily determined, and that in any direction, to a sufficient degree of accuracy, because of the smallness and given position of the base; for, on account of its smallness, all the matter in the pillar may be supposed to be collected into its axis or vertical line erected on the middle of the base, the length of which axis, as the mean altitude of the pillar, is to be estimated
from

from the altitudes of the points in the plan which fall within and near the base of the pillar: then, having given the altitude of this axis, with the position of its base, and the matter supposed to be collected into it, a theorem can easily be given by which the effect of its attraction may be computed. But to retain the proper degree of accuracy in this computation, it is evident that the plan must be divided into a great number of parts, perhaps not less than a thousand for each observatory, in order that they may be sufficiently small, and by this means forming about two thousand of such pillars of matter, whose attractions must be separately computed, as mentioned above. The labour and time necessary for such computation, it is evident, would be very great, perhaps not less than those employed in all the preceding computations of the sections, and all the other points and lines concerned in this business. For this reason I was desirous of obtaining a theorem or method by which the attractions of the small and numerous pillars might be computed with the same degree of accuracy, but with less expence of labour and time than when computed separately as above mentioned. And in this inquiry the success has been equal to my wishes, having at length met with a method by which the business has been effected in perhaps one-fourth or one-fifth of the
time

time that would have been required in the other way. This method I have investigated partly from some hints of the honourable HENRY CAVENDISH, F. R. S. and partly from some of my own, which had been communicated to the Astronomer Royal in the years 1774 and 1775: of which method and its investigation I shall now give some account.

Of all the methods of dividing the plan into a great number of small parts, I have found that to be the most convenient for the computation, in which it is first divided into a number of rings by concentric circles, and these again divided into a sufficient number of parts by radii drawn from the common center, that center being the observatory where the plummet is placed on which the effect of attraction is to be computed. By this means the plan is divided into a great number of small quadrilateral spaces, two of the opposite sides of which are small portions of adjacent circles, and the other two are the intercepted small parts of two adjacent radii, as appears by fig. 1. tab. x. in which, for the present, let the circles and their radii be supposed to be drawn at any distances whatever from each other, till it shall appear from the theorem to be investigated what may be the properest distances and positions of those lines. In this figure A is the observatory, AN the meridian, NAE an

East-and-West line, BCDE one of the little spaces, and F its center or foot of the axis of the pillar whose base is BCDE; the figure AWNEA being a horizontal or level section through the point A. Join A, F, and with the center A describe the middle circle GFH. Let a denote the length of the axis on the point F, or the mean height of the pillar on the base BD; and s = the sine of the angle of elevation of that pillar as observed at A, to the radius 1, or

$s = \frac{a}{\sqrt{a^2 + AF^2}}$. Then will the magnitude of that column

or its quantity of matter be expressed by $\frac{BC+ED}{2} \times BE \times a$,

which is supposed to be all collected into the axis: consequently, if the attraction of each particle of matter be in the reciprocal duplicate ratio of its distance, the attraction of the matter in the pillar, so placed on the plummet at A, in the direction of the meridian AN, will be

$$\frac{BC+ED}{2AF} \times BE \times a \times \frac{s}{a} \times c = \frac{BC+ED}{2AF} \times BE \times sc = \frac{GH}{AF} \times BE \times sc$$

nearly, supposing F to be equally distant from BC and ED, and c the cosine of the angle FAN to the radius 1.

But $\frac{GH}{AF} \times c$ is nearly equal to d the difference of the sines of the angles BAN, CAN, as is thus demonstrated. Draw GK, FL, HM, perpendicular, and GP parallel to AW; and draw the chord GH. Then AK, AM are the sines of the angles GAN, HAN, to the radius AF, their difference being

being $KM = GP$; also FL is the cofine of FAN to the same radius: consequently $GP : FL = d : c$. But the triangles LFA , PCH are equiangular, and therefore $GP : FL = GH : AF$. Consequently $GH : AF = d : c$; or $\frac{GH}{AF} \times c = d$. This equation is accurately true when GH is the chord of the arc; and as the small arc differs insensibly from its chord, the same equation is sufficiently near the truth when GH is the arc itself. Substituting now d instead of the quantity $\frac{GH}{AF} \times c$ in the theorem above, it will become $BE \times d$ for the measure of the attraction of the pillar whose base is BD in the direction AN . Which is as easy and simple an expression for the attraction of a single pillar as can well be desired or expected.

But to make the application of this theorem still more easy to the great number of small pillars concerned in this business, let us suppose BE and d to be constant or invariable quantities, and then it is evident that we shall have nothing more to do but to collect all the s 's or fines of elevation of all the pillars into one sum, and then multiply that sum by the constant quantity $BE \times d$, by which there will be produced the measure of the attraction of all the pillars, or of the whole part of the ground on one side of WE . Now BE will be made to become constant, by making the circles equi-distant from one another,

another, or by taking the radii in arithmetical progression. And d will be constant, by drawing the radii so as to form with AN angles whose sines shall be in arithmetical progression; for then d is the common difference of the sines of those angles. Hence then we are easily led to the best manner of dividing the plan into the small spaces, *viz.* from the center A describe a sufficient number of concentric and equidistant circles; divide the radius AI of any one of them into a sufficient number of equal parts, and from the points of division erect perpendiculars to meet the circle; then through the points of intersection draw radii, and they will divide the circles in the manner required.

In a computation of this kind, we need only calculate the attraction of the matter above the plane or horizon of each observatory, and the attraction of so much matter as is wanting to fill up the vacuity below that plane lying between it and the surface of the lower part of the hill. For the South observatory, the attraction of the Southern parts that are above it must be subtracted from that of the Northern parts, to obtain the attraction of the whole towards the North; that is, the Southern elevations are negative, and the Northern ones affirmative. The contrary names take place with respect to the depressions, or the vacuities below the plane of the observatory; for if

the whole space below this horizontal plane were full of matter to an equal extent both ways, its attraction need not be computed, as those on the contrary sides would mutually balance each other; but since there are unequal vacuities on each side, it is evident, that the attraction of the matter that might be contained in them must be deducted from the other two equal quantities, to leave the real attraction of those two sides; then subtracting the remainder to the South side from that of the Northern side, there will at last remain the joint effect of all the matter below the plane in the Northern direction: but as the one remainder is to be subtracted from the other, the two equal quantities may be omitted in both, and only the effects of the vacuities brought into the account, which being twice subtracted, their signs become contrary to those of the parts above the horizontal plane; that is, the effect of the Southern vacuity is affirmative, and that of the Northern one negative. But for the Northern observatory, when the attraction towards the South is to be found, the contrary names take place; that is, in the elevations the Southern parts are affirmative, and the Northern parts negative; but in the vacuities or depressions, the Northern parts are affirmative, and the Southern ones negative.

According

According to the foregoing method the plan of the ground was divided into 20 rings by equidistant concentric circles, described about each observatory as a center; and each quadrant was divided into 12 parts or sectors by lines forming, with the meridian, angles whose sines are in arithmetical progression; by which means the space in each quadrant was divided into 240 small parts, making almost a thousand of such parts in the whole round for each observatory, or near 2000 for the two observatories. This was judged to be a sufficiently great number of parts to afford a very considerable degree of accuracy; or at least that number was as great, and the parts as small, as was well consistent with the degree of accuracy afforded by the number of points whose relative altitudes had been determined.

In this division the common breadth of the rings, or the common difference of the radii, is $666\frac{2}{3}$ feet; and the common difference of the sines of the angles formed by the radii and the meridian is $\frac{1}{12}$ th of the radius; and consequently, those angles are expressed in degrees and minutes as here follows, *viz.* $4^{\circ} 47'$, $9^{\circ} 36'$, $14^{\circ} 29'$, $19^{\circ} 28'$, $24^{\circ} 37'$, $30^{\circ} 0'$, $35^{\circ} 41'$, $41^{\circ} 48\frac{1}{2}'$, $48^{\circ} 35'$, $56^{\circ} 26\frac{1}{2}'$, $66^{\circ} 26\frac{1}{2}'$, $90^{\circ} 0'$.

Tab. IX. contains a small plan of the principal and most central part of the ground, accurately divided in the

above manner for one of the observatories, namely, the Northern one, with the places of all or most of the points which fall within this part of the ground, accurately laid down and marked with dots, as also such of the included letters as have been before mentioned in this paper.

In this plate RABCD, &c. is the chain of stations around the hill; N and K are the West and East cairns on the extremities of the ridge of the hill; o the Southern observatory, and p the Northern one. Of this kind were made two large plans, one divided for each observatory, from which were estimated the mean altitudes of the pillars erected on the spaces into which they are divided.

These altitudes are easily estimated when several of the points fall near and in the small spaces or bases, especially when they are near the middle of them; but, numerous as the points are, there are evidently many bases in which none at all are contained, nor even near them. This circumstance at first gave me much trouble and dissatisfaction, till I fell upon the following method by which the defect was in a great measure supplied, and by which I was enabled to proceed in the estimation of the altitudes both with much expedition and a considerable degree of accuracy. This method was the connecting together by a faint line all the points which were
of

of the same relative altitude: by so doing, I obtained a great number of irregular polygons lying within, and at some distance from, one another, and bearing a considerable degree of resemblance to each other: these polygons were the figures of so many level or horizontal sections of the hills, the relative altitudes of all the parts of them being known; and as every base or little space had several of them passing through it, I was thereby able to determine the altitude belonging to each space with much ease and accuracy. In this estimation I could generally be pretty sure of the altitude to within ten feet, and often within five, which on an average might be about the 100th part of the whole altitude; and when we consider that the number of such estimated altitudes is very great, and that it is probable the small errors among them would nearly balance one another, the defect of those that might be reckoned too little being compensated by the excess in those which might be taken too great, we need not hesitate to pronounce, that the error arising from the estimation of the altitudes is probably still much less than that part.

It was necessary to determine these altitudes of the pillars, in order to compute the sines of the angles of elevation subtended by them, as the theorem requires the use of these sines; and the very easy method used in

deducing the latter from the former shall be explained after we have, as below, registered the altitudes of all the pillars as they were computed. This register consists of sixteen tables, namely four quadrants of spaces in the altitudes, and four in the depreffions, for each observatory, as specified in the titles of them. The numbers are feet, like all the other dimensions. The numbers on the same horizontal line from left to right are such as are all in the same ring; and those in one and the same vertical column are in the same sector, or between the same two radii; the number of the ring, counted from the common center, is written in the left-hand margin; and the number of the vertical column or distance of the space or sector from the meridian, at the top; also the radius of each ring, that is, the line from the common center to the middle of the ring is written on the same line with it, in the right-hand margin. It may be further remarked, that in such little spaces as were cut through by the boundary line between elevations and depreffions, thereby making but a part of such spaces in each of those denominations, each space was accounted as a whole one; but then the mean altitude or depreffion in each part was diminished in the proportion of the whole space to the part of it so included in the boundary. The altitudes and depreffions are put down first with respect to

the Southern observatory o, and then for the Northern observatory p; and in each, the altitudes are placed first.

1. Altitudes above o in the N.W. quarter.													
Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radii.
1	215	215	215	215	210	205	200	190	170	145	120	75	333 ¹
2	605	610	605	600	595	590	580	570	510	450	350	200	1000
3	965	1005	1010	1010	1020	1050	1040	900	810	600	415	220	1667
4	670	680	700	780	860	930	1040	1090	1100	760	480	210	2333
5	280	310	370	450	560	700	830	960	1180	890	545	200	3000
6	20	50	100	110	250	380	525	710	890	950	605	110	3667
7					10	70	120	415	620	780	600	120	4333
8							15	95	390	610	480	35	5000
9									135	310	220	5	5667
10										40	50		6333

2. Altitudes above o in the N.E. quarter.													
Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radii
1	210	205	205	200	195	185	170	155	140	125	105	70	333 $\frac{1}{2}$
2	550	545	540	530	520	510	500	465	430	370	270	130	1000
3	910	840	825	815	800	760	720	680	635	590	500	200	1667
4	645	640	635	640	645	650	675	715	730	700	580	300	2333
5	265	255	265	285	310	350	390	450	460	500	600	280	3000
6	10	12	20	65	100	130	160	180	180	320	460	300	3667
7								15	55	110	150	250	4333
8												50	5000

3. Altitudes above o in the S.W. quarter.													
Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radii
1												10	333
2												15	1000
3												5	1667
12												40	7667
13												60	8333
14												100	9000
15												200	9667
16	40	120	200	250	280	280	260	170	60			300	10333
17	160	270	360	400	440	450	450	390	270	30		500	11000
18	310	420	500	580	620	650	660	630	500	290		700	11667
19	440	540	620	680	740	800	800	780	600	400	60	800	12333
20	550	650	750	800	900	950	960	960	800	500	200	900	13000

4. Altitudes above o in the S.E. quarter.

Rings.	1	2	3	4	5	6	7	8		Radil.
16	10									10333
17	80	60	50	50	10					11000
18	220	200	180	130	70	30				11667
19	340	300	260	240	170	120	20			12333
20	450	410	380	330	260	180	100	20		13000

5. Depressions below o in the N.W. quarter.

Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radil.
6	70	40	15	5								15	3667
7	250	240	200	150	60	30						40	4333
8	460	450	430	390	280	200	80	10				80	5000
9	700	700	680	630	520	450	340	170	15			120	5667
10	840	830	800	780	650	600	520	380	180	40	70	220	6333
11	960	920	880	850	750	650	630	550	350	250	300	430	7000
12	1100	1000	950	900	820	780	780	780	580	530	500	560	7667
13	1130	1080	980	880	840	800	830	860	780	690	630	640	8333
14	1180	1100	1000	900	900	900	910	940	870	800	700	500	9000
15	1180	1100	1100	1080	1040	1050	1060	1070	1000	870	730	300	9667
16	1100	1100	1100	1100	1100	1140	1150	1150	1120	990	760	160	10333
17	1100	1100	1100	1130	1180	1200	1200	1200	1180	1080	700	80	11000
18	1100	1100	1150	1200	1200	1150	1100	1100	1200	1180	700	100	11667
19	1100	1120	1220	1230	1260	1200	1200	1200	1300	1240	620	60	12333
20	1120	1220	1320	1360	1390	1390	1390	1340	1440	1300	620	50	14000

6. Depressions below o in the N.E. quarter.

Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radil.
6	70	60	30	10								10	3667
7	260	240	200	150	110	80	30	40	30	10		10	4333
8	450	440	400	350	280	180	100	180	180	190	190	40	5000
9	700	690	680	610	520	400	200	240	290	350	330	200	5667
10	850	870	890	860	770	620	440	300	380	500	450	370	6333
11	1020	1060	1070	1050	980	860	700	520	400	650	600	530	7000
12	1140	1160	1180	1160	1140	1080	950	840	620	720	850	700	7667
13	1200	1190	1200	1220	1240	1250	1160	1050	900	840	950	880	8333
14	1230	1130	1050	1050	1100	1220	1260	1220	1070	950	1020	990	9000
15	1100	960	900	850	900	1100	1230	1210	1170	1060	1090	1100	9667
16	970	860	880	780	780	900	1120	1180	1200	1180	1160	1150	10333
17	970	800	760	750	750	780	1000	1200	1300	1240	1200	1100	11000

7. Depressions below o in the S.W. quarter.

Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radii
1	165	165	160	155	150	140	130	120	110	90	50	10	333
2	400	390	380	370	350	330	300	270	240	210	160	60	1000
3	600	580	560	530	500	570	540	400	370	340	280	100	1667
4	740	720	700	670	640	610	580	530	490	440	370	160	2333
5	800	800	800	770	740	710	660	610	570	510	440	230	3000
6	780	790	780	770	780	800	790	700	650	590	510	320	3667
7	700	710	720	730	750	750	750	750	730	670	600	400	4333
8	580	590	600	610	640	660	700	720	730	730	700	520	5000
9	490	490	490	480	490	510	600	650	660	690	580	450	5667
10	470	460	420	400	420	420	440	490	580	590	560	430	6333
11	340	340	340	340	340	330	350	390	450	480	380	370	7000
12	210	220	230	250	250	250	280	310	340	370	250	200	7667
13	160	150	140	120	130	150	200	230	280	290	230	110	8333
14	110	90	60	20	20	20	70	150	230	240	150	90	9000
15	50	20						40	140	180	90	50	9667
16									30	90	70	20	10333

8. Depressions below o in the S.E. quarter.

Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radii
1	165	165	160	155	150	140	130	120	110	100	95	40	333
2	400	400	400	400	400	390	380	360	330	300	250	110	1000
3	600	610	610	610	610	600	600	580	550	500	440	200	1667
4	760	750	740	740	740	730	720	710	680	640	560	300	2333
5	800	800	800	800	800	800	800	800	790	740	660	400	3000
6	780	780	780	780	780	790	800	840	880	850	750	470	3667
7	700	690	680	670	670	680	620	720	820	900	770	520	4333
8	580	570	570	570	570	580	590	600	660	800	800	600	5000
9	490	490	490	490	490	490	490	500	520	700	880	600	5667
10	470	460	450	440	430	420	410	430	470	530	880	670	6333
11	340	330	320	320	320	320	330	350	420	500	780	780	7000
12	210	200	200	200	210	220	240	280	390	480	680	900	7667
13	120	120	130	130	140	150	180	230	300	450	600	990	8333
14	110	110	110	120	130	150	160	200	280	440	580	930	9000
15	70	70	70	70	90	120	140	170	240	420	570	900	9667
16	10	20	30	40	50	80	120	160	220	400	550	1000	10333
17						40	90	140	200	340	540	950	1100
18						5	40	110	170	300	500	850	1166
19									50	150	280	470	1233
20									20	150	250	400	1300

9. Altitudes above p in the N.W. quarter.

Rings.		12	Radii.
4		10	2333
5		15	3000
6		15	3667
7		15	4333
8		15	5000

10. Altitudes above p in the N.E. quarter.

Rings		12	Radii
1		10	333 $\frac{1}{3}$
2		10	1000
3		15	1667
4		60	2333
5		40	3000
6		5	3667

11. Altitudes above p in the S.W. quarter.

Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radii
1	110	110	105	105	100	100	95	95	90	85	80	35	333 $\frac{1}{3}$
2	340	330	320	310	300	290	280	270	240	210	170	90	1000
3	660	660	660	660	660	650	620	590	570	510	370	170	1667
4	1020	1030	1040	1050	1060	1070	1030	990	910	800	660	270	2333
5	1020	1110	1280	1270	1320	1330	1310	1280	1270	1170	910	460	3000
6	670	770	810	900	930	950	1020	1070	1150	1270	1100	600	3667
7	280	340	420	480	540	570	620	670	720	880	1050	660	4333
8	20	50	90	140	210	290	350	420	490	570	700	570	5000
9							120	210	270	320	370	270	5667
10									90	150	180	20	6333
15									140	170			9667
16									140	470	30		10333
17									130	500	170	170	11000
18								20	170	600	400	400	11667
19	150	150	140	120	90	60	30	70	170	500	500	500	12333
20	160	170	210	220	200	170	100	130	170	400	600	600	13000

12. Altitudes above p in the S.E. quadrant.													
Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radius
1	110	110	105	105	100	100	95	95	90	85	80	35	333½
2	340	330	320	310	300	290	280	270	240	210	170	90	1000
3	660	640	620	600	570	540	510	480	440	380	300	160	1667
4	1000	980	950	910	870	810	730	670	540	460	330	170	2333
5	1020	1020	1020	1030	1030	1020	970	770	570	470	390	130	3000
6	670	710	770	810	840	860	910	890	720	650	400	30	3667
7	290	320	360	390	470	590	700	750	780	600	280		4333
8				20	70	170	250	420	630	550	170		5000
9								110	360	420	170		5667
10									70	180	120		6333
18		20	40	50	40								11667
19	100	100	100	100	80	30							12333
20	130	130	130	120	110	80							13000

13. Depressions below p in the N.W. quarter.													
Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radius
1	100	95	90	85	80	75	70	60	50	40	30	15	333½
2	390	380	360	330	310	290	270	240	210	180	150	60	1000
3	520	510	500	490	480	470	450	430	410	370	270	80	1667
4	650	640	620	610	590	570	550	530	500	460	390	90	2333
5	830	820	760	720	690	660	630	590	560	500	380	130	3000
6	880	860	850	790	730	700	670	640	580	480	340	280	3667
7	910	900	860	830	790	720	630	620	540	550	440	185	4333
8	930	890	840	800	830	710	610	610	580	530	520	430	5000
9	830	830	830	830	830	830	760	700	670	620	600	330	5667
10	730	740	755	770	785	800	815	830	780	750	720	460	6333
11	730	750	780	800	830	860	860	880	880	860	820	530	7000
12	750	770	810	860	910	930	950	960	950	930	880	560	7067
13	770	840	910	950	990	1030	1050	1050	1030	980	950	650	8333

14. Depressions below p in the N.E. quarter.

Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radii.
1	80	75	70	65	60	55	50	45	40	35	30	20	333 $\frac{1}{3}$
2	330	325	320	300	280	260	240	220	190	150	110	30	1000
3	520	515	505	490	475	460	440	420	400	330	240	60	1667
4	660	675	690	700	700	690	660	620	560	470	350	25	2333
5	840	840	840	840	840	830	820	770	720	620	440	100	3000
6	860	880	900	920	930	930	910	870	830	740	570	240	3667
7	920	920	920	880	880	900	930	940	930	840	680	610	4333
8	920	840	780	780	740	720	770	870	920	970	900	630	5000
9	720	670	600	600	600	560	580	670	850	950	940	600	5667
10	700	620	520	500	500	500	500	520	720	920	960	650	6333
11	700	600	600	600	620	600	580	560	540	840	920	770	7000
12	720	700	680	700	720	740	700	740	570	800	920	820	7667
13	720	720	720	720	700	700	720	720	620	820	900	920	8333

15. Depressions below p in the S.W. quarter.

Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radii.
9	230	140	100	70	40	40							5667
10	400	340	280	230	190	150	110	30					6333
11	500	470	410	340	290	260	230	200	150	30		140	7000
12	500	510	510	490	410	370	330	310	350	280	150	260	7667
13	480	500	500	510	500	460	430	260	150	230	280	360	8333
14	370	390	400	430	450	450	400	210	10	110	280	230	9000
15	260	260	250	260	330	330	310	130			130	130	9667
16	200	200	150	160	170	220	230	130					10333
17	130	130	90	80	90	110	140	30					11000
18	10	20	30	10	10	20	30						11667

16. Depreffions below P in the S.E. quarter.													
Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radii
7												80	4333
8	20	30	30									210	5000
9	260	290	290	280	240	150	30					270	5667
10	420	440	450	440	400	370	270	140				330	6333
11	530	540	560	560	550	480	430	330	150	40	40	430	7000
12	500	510	520	550	630	600	500	430	290	230	200	630	7667
13	450	430	420	410	430	570	630	530	430	480	340	710	8333
14	360	330	310	290	280	330	510	670	590	630	500	830	9000
15	240	230	220	200	180	100	330	530	770	760	710	870	9667
16	180	160	150	130	110	140	230	330	630	830	790	880	10333
17	110	80	50	40	30	90	190	280	500	860	830	860	11000
18	10					10	150	260	400	760	830	760	11667
19							70	230	330	600	770	630	12333
20							10	180	290	530	690	530	13000

It remains now to find the fines of the vertical angles subtended by all the foregoing altitudes and depreffions, since the fum of these fines is what we are in quest of. Each altitude or depreffion is the perpendicular of a right-angled triangle, of which the given radius standing on the same line with it in the right-hand margin is the base, or other side about the right-angle; and by the resolution of the right-angled triangle, for each perpendicular, the same number of corresponding fines will be found. But with such *data* the tangent of the angle is much easier to be found than the fine, and the analogy for that purpose is this, as the base : to the perpendicular :: 1 (radius) : the tangent required, which will therefore

fore be found by barely dividing the given perpendicular by the base; and if we find this number in its proper column in a table of sines and tangents, on the same line with it, in the column of sines will be found the sine of the angle required. This seems to be the easiest way of resolving all the triangles when computed separately. But as the labour would be very great in performing so many hundreds of arithmetical divisions, &c. either by logarithms, or by the natural numbers, instead of it, the following method, proposed by the Hon. Mr. CAVENDISH, was adopted, as being a much more expeditious way of obtaining the sum of the sines required. This method consists in finding, in a very easy manner, the difference between each tangent and its corresponding sine, from the given base and perpendicular, and then, subtracting the sum of all the differences from the sum of the tangents, there remains the sum of the sines. Several advantages attend this method of proceeding: for, to find the tangents we need not divide every perpendicular separately by its corresponding base, but add together all the perpendiculars that are on the same line, and divide their sum by their common base, which is the radius of the middle of the ring, and is placed on the same line with them towards the right-hand; for thus we shall have little more than a twelfth part of the number of divisions

divisions to perform: also a great part of the tangents are so small that they do not at all differ from their corresponding sines in the number of decimals that it is necessary to continue the computations to, in all which cases the trouble of finding the differences is saved; and those differences which it is necessary to compute, are very readily found by inspection on a peculiar kind of sliding rule, which was constructed for this purpose, and of which I shall here give a short description.

This rule (the figure of which is represented tab. x. fig. 2.) consists of three columns; one marked *AF* or base, which is moveable by sliding it up or down by the side of the other two which are fixed; of these two the one contains the perpendicular altitudes or depressions, and the other the differences between the sines and tangents to the radius 1. To construct the numbers on this rule; form a series of logarithmic tangents in arithmetical progression, of which the first term is 9°000, and the common difference 0°25; take out from a table the corresponding natural tangents, and place them in the first and second columns of base and perpendicular, and the difference between the natural sine and natural tangent in the last column, marked *Diff*. To make use of this scale; look out any base and its corresponding perpendicular in their proper columns, that is, any radius and its

cor-

corresponding altitude or depreſſion in the ſixteen foregoing tables, without regarding the number of places they contain, and bring them to correſpond; then, if they conſiſt of the ſame number of places, the lower index on the ſlider or firſt column, or that anſwering to 1000, points to the true difference between the ſine and tangent in the laſt column; but if the number of places in the baſe exceed that in the perpendicular by one, the upper index 100 muſt be uſed. And in this manner were computed all the differences which were neceſſary to be found, and placed in their proper ſquares formed by the meeting of the horizontal and vertical lines, or rings and ſectoral ſpaces, in the following ſet of ſixteen tables, which correſpond to the foregoing ſet of ſixteen, each to each, according to the number of them, and marked at the tops with the numbers 1, 2, 3 &c. to 12 for the ſectoral ſpaces, and with the number of the rings on the left-hand margin. Alſo, in the column immediately after the number of the ring are placed the radii which formed the laſt column in the preceding tables; then, in the third column, are placed the ſums of the altitudes and depreſſions found in each line of the former tables; and, in the next column, the quotients found by dividing the numbers in the third by thoſe in the ſecond column; theſe quotients are the ſums of the tangents
belonging

belonging to each line or ring, which being all added together, their total is placed at the bottom of the column: after this follow the twelve columns of differences before mentioned, which are succeeded by one more column containing the sums of each line of these differences, which sums being added together, their total is placed at the bottom of them; and this total is the sum of all the differences between the fines and the tangents, and it is therefore subtracted from the total of the tangents in the fourth column, when there remains the sum of the fines as required.

1. For the sum of the fines of alt. above 0 in the N.W. quarter.																
Rings.	Radii.	Sum of Perpen.	$3 \div 2 =$ Sum of Tang.	1	2	3	4	5	6	7	8	9	10	11	12	Sum of Diff.
1	333 $\frac{1}{2}$	2175	6'525	103	103	103	103	97	97	86	75	55	36	20	5	883
2	1000	6265	6'265	88	90	88	85	84	82	78	75	56	40	20	4	790
3	1667	10045	6'027	79	86	87	87	88	97	93	64	48	21	7	1	758
4	2333	9300	3'986	12	12	13	17	23	27	39	43	45	16	4		251
5	3000	7275	2'425		1	1	2	3	6	11	16	27	13	3		83
6	3667	4695	1'280						1	1	3	7	9	2		23
7	4333	2735	0'631								1	1	3	1		6
8	5000	1625	0'325										1	1		2
9	5667	670	0'118													
10	6333	60	0'009													
			27'591 = sum of tangents.												Sum of diff. 2'796	
			2'796 = sum of the diff.													
			24'795 = sum of the fines of alt. above 0 in the N.W. quarter.													

2. For the sum of the fines above 0 in the N.E. quarter.

Rings.	Radii.	Sum of Perp.	$3 \div 2 =$ Sum of Tang.	1	2	3	4	5	6	7	8	9	10	11	12	Sum of Diff.
1	333 $\frac{1}{2}$	1965	5.895	97	92	92	87	80	71	56	43	33	24	15	5	.695
2	1000	5360	5.360	68	67	65	62	59	56	53	43	35	23	9	1	541
3	1667	8275	4.965	66	53	51	49	47	41	36	31	25	20	13	1	433
4	2333	7555	3.238	10	10	10	10	10	11	12	14	15	13	8	1	124
5	3000	4410	1.470						1	1	1	2	2	2	4	13
6	3667	1937	.528										1	1		2
7	4333	580	.134													
8	5000	50	.010													
			21.600 = sum of the tangents.													1.808
			1.808 = sum of the differences.													
			19.792 = sum of the fines.													

3. For the sum of the fines above 0 in the S.W. quarter.

1	333 $\frac{1}{2}$	10	0.030
2	1000	15	15
3	1667	5	3
12	7667	40	5
13	8333	60	7
14	9000	100	11
15	9667	200	21
16	10333	1960	190
17	11000	3720	338
18	11667	5770	495
19	12333	7260	573
20	13000	8920	686

2.374 = sum of the tangents, or sum of the fines, as the diff. between them are nothing in this quadrant.

4. For the sum of the fines above 0 in the S.E. quarter.

16	10333	10	0.001
17	11000	230	21
18	11667	830	71
19	12333	1450	118
20	13000	2130	164

0.375 = sum of the tangents, or of the fines, the diff. being nothing.

5. For the fines below 0 in the N.W. quarter.

Rings.	Radii.	Sum of Perp.	$3 \div 2 =$ Sum of Tang.	1	2	3	4	5	6	7	8	9	10	11	12	Sum of Diff.
6	3667	145	0.040													
7	4333	970	224													
8	5000	2370	474													
9	5667	4325	763	1	1	1	1	1								5
10	6333	5910	933	1	1	1	1	1	1							6
11	7000	7520	1.074	1	1	1	1	1	1	1						7
12	7667	9280	1.210	2	1	1	1	1	1	1	1					9
13	8333	10140	1.219	2	1	1	1	1	1	1	1	1				10
14	9000	10700	1.078	1	1	1	1	1	1	1	1	1	1			9
15	9667	11580	1.198	1	1	1	1	1	1	1	1	1	1	1		10
16	10333	11970	1.159	1	1	1	1	1	1	1	1	1	1			9
17	11000	12250	1.105	1	1	1	1	1	1	1	1	1	1	1		10
18	11667	12280	1.052	1	1	1	1	1	1	1	1	1	1	1		10
19	12333	12750	1.034			1	1	1	1	1	1	1	1	1		8
20	13000	13940	1.072			1	1	1	1	1	1	1	1	1		8

13.635 = sum of the tangents.

.101 = sum of the differences.

13.534 = sum of the fines.

.101

6. For the sum of the fines below 0 in the N.E. quarter.

				1	2	3	4	5	6	7	8	9	10	11	12	
6	3667	180	0.049													
7	4333	1160	268													
8	5000	2980	596													
9	5667	5210	919	1	1	1	1	1								5
10	6333	7300	1.153	1	1	1	1	1	1							6
11	7000	9440	1.349	1	2	2	2	1	1	1						10
12	7667	11460	1.495	2	2	2	2	2	1	1	1		1	1		15
13	8333	13080	1.570	1	1	1	1	2	2	1	1	1	1	1	1	14
14	9000	13290	1.477	1	1	1	1	1	1	1	1	1	1	1	1	12
15	9667	12670	1.311	1	1	1		1	1	1	1	1	1	1	1	11
16	10333	12160	1.177		1		1		1		1		1		1	6
17	11000	11850	1.077			1			1		1	1		1	1	6

12.441 = sum of the tangents.

.085 = sum of the differences.

12.356 = sum of the fines.

.085

8. For the fum of the fines below o in the S.E. quarter.

Rings.	Radii.	Sum of Dep.	$3 \div 2 =$ Sum of Tang.	1	2	3	4	5	6	7	8	9	10	11	12	Sum of Diff.
1	333 $\frac{1}{3}$	1510	4'530	51	51	47	43	40	35	27	21	17	13	11	1	357
2	1000	4120	4'120	28	28	28	28	28	27	25	21	17	13	8	1	252
3	1667	6510	3'906	21	21	21	21	21	21	21	19	16	13	9	1	205
4	2333	8070	3'459	16	16	15	15	15	15	14	14	13	10	6	1	150
5	3000	8990	2'997	9	9	9	9	9	9	9	9	9	8	5	1	95
6	3667	9280	2'531	5	5	5	5	5	5	5	5	6	6	4	1	57
7	4333	8440	1'948	2	2	2	2	2	2	2	2	3	4	3	1	27
8	5000	7490	1'498	1	1	1	1	1	1	1	1	1	2	2	1	14
9	5667	6630	1'170			1		1				1	1	1	1	6
10	6333	6070	958			1			1			1		1	1	5
11	7000	5110	730				1				1		1	1	1	5
12	7667	4210	549						1				1		1	3
13	8333	3540	425								1			1	1	3
14	9000	3370	374								1			1	1	3
15	9667	3020	313										1		1	2
16	10333	2680	259											1	1	2
17	11000	2310	210												1	1
18	11667	1975	169												1	1
19	12333	1730	140												1	1
20	13000	1520	117												1	1
			30'403 = fum of the tangents.													1'190
			1'190 = fum of the differences.													
			29'213 = fum of the fines.													

9. For the fum of the fines above p in the N.W. quarter.

4	2333	10	'004	
5	3000	15	5	
6	3667	15	4	
7	4333	15	3	
8	5000	15	3	

0 019 = fum of the tangents, or fum of the fines, as they have no difference in this quadrant.

10. For the fum of the fines above p in the N.E. quarter.

1	333 $\frac{1}{3}$	10	'030	
2	1000	10	10	
3	1667	15	9	
4	2333	60	26	
5	3000	40	13	
6	3667	5	2	

0'090 = fum of the tangents, or fum of the fines, they being equal in this quadrant.

I 1. For the sum of the fines above P in the S.W. quarter.

Rings.	Radii.	Sum of Alt.	$3\frac{1}{2} =$ Sum of Tang.	1	2	3	4	5	6	7	8	9	10	11	12	Sum of Diff.
1	333 $\frac{1}{2}$	1110	3'330	17	17	15	15	13	13	12	12	10	8	6	1	139
2	1000	3150	3'150	18	17	16	14	13	12	11	10	6	5	2	1	125
3	1667	6780	4'068	28	28	28	28	27	23	20	18	14	5	1		248
4	2333	10930	4'684	37	38	39	40	41	42	38	34	26	18	7	1	361
5	3000	13740	4'580	18	23	34	33	38	39	37	34	33	26	14	2	331
6	3667	11240	3'065	3	5	5	6	6	7	14	16	19	21	14	2	118
7	4333	7230	1'668			1	1	1	1	1	2	2	4	5	2	20
8	5000	3900	780							1		1	1	1	1	5
9	5667	1560	275											1		1
10	6333	440	70													
15	9667	310	32													
16	10333	640	62													
17	11000	970	88											1		1
18	11667	1590	137											1		1
19	12333	2480	201												1	1
20	13000	3130	241											1	1	2

26'431 = sum of the tangents.

1'353

1'353 = sum of the differences.

25'078 = sum of the fines.

I 2. For the sum of the fines above P in the S.E. quarter.

				1	2	3	4	5	6	7	8	9	10	11	12	
1	333 $\frac{1}{2}$	1110	3'330	17	17	15	15	13	13	12	12	10	8	6	1	139
2	1000	3150	3'150	18	17	16	14	13	12	11	10	6	5	2	1	125
3	1667	5890	3'534	28	26	23	21	18	16	14	12	9	5	2	1	175
4	2333	8420	3'609	34	33	30	26	24	18	15	12	6	4	1		203
5	3000	9440	3'147	18	18	18	18	18	18	16	9	3	2	1		139
6	3667	8260	2'253	3	4	5	5	5	6	8	7	4	3	1		51
7	4333	5530	1'276			1		1	1	2	3	3	1			12
8	5000	2280	456							1	1	1				3
9	5667	1060	187									1				1
10	6333	370	58													
18	11667	150	13													
19	12333	510	42													
20	13000	700	54													

21'109 = sum of the tangents.

0'848

0'848 = sum of the differences.

20'261 = sum of the fines.

13. For the fum of the fines below p in the N.W. quarter.

Rings.	Radii.	Sum of Dep.	$3\frac{1}{2} =$ Sum of Tang.	1	2	3	4	5	6	7	8	9	10	11	12	Sum of Diff.
1	333 $\frac{1}{2}$	790	2.370	13	12	9	8	7	5	5	3	2	1	1		066
2	1000	3170	3.170	26	25	21	17	14	12	9	6	5	3	2		140
3	1667	4980	2.988	15	14	13	12	12	11	9	8	7	5	2		108
4	2333	6200	2.657	10	10	9	9	8	7	6	5	5	4	2		75
5	3000	7270	2.423	10	10	8	7	6	5	5	4	3	2	1		61
6	3667	7800	2.127	7	6	5	5	4	3	3	3	2	1	1		40
7	4333	7975	1.840	5	4	4	3	3	2	2	1	1	1	1		27
8	5000	8280	1.656	3	3	2	2	2	1	1	1	1	1	1		17
9	5667	8660	1.528	2	1	2	1	2	1	1	1	1	1	1		14
10	6333	8935	1.411	1	1	1	1	1	1	1	1	1	1	1		11
11	7000	9580	1.369	1		1		1	1	1	1	1	1	1		9
12	7667	10280	1.341	1		1		1	1	1	1	1	1	1		9
13	8333	11200	1.344			1	1	1	1	1	1	1	1	1		10
																26.224 = fum of the tangents.
																0.587 = fum of the differences.
																25.637 = fum of the fines.

14. For the fum of the fines below p in the N.E. quarter.

				1	2	3	4	5	6	7	8	9	10	11	12	
1	333 $\frac{1}{2}$	625	1.875	7	5	5	4	3	2	2	1	1	1	1		032
2	1000	2755	2.755	17	16	15	13	11	9	6	5	3	2	1		98
3	1667	4855	2.913	15	14	13	12	11	10	9	8	6	4	2		104
4	2333	7000	3.000	11	12	13	13	13	13	11	9	6	4	2		107
5	3000	8500	2.833	11	11	11	11	11	10	10	9	7	5	2		98
6	3667	9580	2.613	6	7	7	8	8	8	8	6	5	4	2		69
7	4333	10350	2.389	5	5	5	4	4	4	5	5	5	4	2	1	49
8	5000	11840	2.368	3	2	2	2	2	1	2	3	3	3	3	1	27
9	5667	8400	1.482	1	1	1	1	1	1		1	2	2	2	1	14
10	6333	7610	1.202	1	1		1		1		1	1	1	1	1	9
11	7000	7930	1.133	1		1		1		1			1	1	1	7
12	7667	8810	1.149	1		1		1		1			1	1	1	8
13	8333	8980	1.078		1		1		1		1		1	1	1	7
																26.790 = fum of the tangents.
																6.29 = fum of the differences.
																26.161 = fum of the fines.

15. For the sum of the fines below p in the S.W. quarter.

Rings.	Radii.	Sum of Dep.	$3 \div 2 =$ Sum of Tang.	1	2	3	4	5	6	7	8	9	10	11	12	Sum of Diff.
9	5667	620	051													
10	6333	1730	273			1										001
11	7000	3030	433	1						1						2
12	7667	4470	583		1						1					2
13	8333	4600	559				1				1					2
14	9000	3730	414				1							1		2
15	9667	2390	247							1						1
16	10333	1460	141								1					1
17	11000	800	73													
18	11667	130	11													
			2.785 = sum of the tangents.													011
			011 = sum of the differences.													
			2.774 = sum of the fines.													

16. For the sum of the fines below p in the S.E. quarter.

				1	2	3	4	5	6	7	8	9	10	11	12	
7	4333	80	018													
8	5000	290	58													
9	5667	1810	319			1										001
10	6333	3280	518			1				1						2
11	7000	4640	663		1				1						1	3
12	7667	5590	729			1			1						1	3
13	8333	5830	700				1			1					1	3
14	9000	5700	633				1				1			1		3
15	9667	5240	542							1			1		1	3
16	10333	4560	441								1			1		2
17	11000	3920	356									1			1	2
18	11667	3180	273											1		1
19	12333	2630	213											1		1
20	13000	2230	172											1		1
			5.635 = sum of the tangents.													025
			025 = sum of the differences.													
			5.610 = sum of the fines.													

Having now obtained the sums of the sines for the several quadrants, the next business is to collect them together, and deduct the negatives from the affirmatives. And this may be done either for each observatory separately, or for both together. I shall do them separately, in order thereby to discover also the ratio of their effects.

And, first, for the Southern observatory o.

Affirmatives.		Negatives.	
1 . . 24.795 N.W.	} Alt.	3 . . 2.374 S.W.	} Alt.
2 . . 19.792 N.E.		4 . . 0.375 S.E.	
7 . . 24.806 S.W.	} Dep.	5 . . 13.534 N.W.	} Dep.
8 . . 29.213 S.E.		6 . . 12.356 N.E.	
98.606 = sum of affirm.		28.639 sum.	
28.639 = sum of negat.			

69.967 = effective sum of the sines for o.

Secondly, for the Northern observatory p.

Affirmatives.		Negatives.			
11 .. 25.078 S.W.	} Alt.	9 .. 0.019 N.W.	} Alt.		
12 .. 20.261 S.E.		10 .. 0.090 N.E.			
13 .. 25.637 N.W.	} Dep.	15 .. 2.774 S.W.	} Dep.		
14 .. 26.161 N.E.		16 .. 5.610 S.E.			
97.137 = sum of affirm.		8.493			
8.493 = sum of negat.					

88.644 = effective sum of the sines for p.

69.967 = the same for o.

158.611 = the sum of the sines for both observ.

From these numbers it appears, that the effect of the attraction at the Northern observatory is to that at the Southern one, nearly as 70 is to 89, or as 7 to 9 nearly. This difference is to be attributed chiefly to the effect of the hills on the South of the Southern observatory, which were considerably greater and nearer to it than those on the back of the Northern observatory. For although the Southern observatory was placed 273 feet above the level of the Northern one, which removed it considerably more above the center of gravity of the hill than the latter was, it was at the same time placed considerably nearer than the other to the middle in a horizontal direction; so that probably the one difference nearly balances the other; and accordingly we find that the sum of the affirmative altitudes for o is 44.587, and of those for p 45.339, which differ by only a 45th part nearly.

It only remains now to multiply the sum of the sines by the common breadth of the rings, and by the common difference of the sines of the angles made by the meridian and the several radii. It has already been observed, that the former is $666\frac{2}{3}$, and the latter $\frac{1}{12}$; therefore $\frac{1}{12} \times 666\frac{2}{3} = \frac{2000}{36} = \frac{500}{9}$ is their product: consequently, $158.611 \times \frac{500}{9} = 8811\frac{2}{3}$ nearly, is the sum of the two opposite attractions made by the hill, &c. at the two observatories.

In order now to compare this attraction with that of the whole earth, this body may be considered as a sphere, and the observatories as placed at its surface; since the very small differences of these suppositions from the truth, are of no consequence at all in this comparison. Now the attraction of a sphere, on a body at its surface, is known to be $= \frac{2}{3}cd$, where d is = the diameter of the sphere, and $c = 3 \cdot 1416$ = the circumference of the circle of which the diameter is 1. But cd is = the circumference of the circle to the diameter d ; and therefore the attraction of a sphere will be expressed by barely $\frac{2}{3}$ of its circumference; which is a theorem well adapted to the computation in hand. The length of a degree in the mean latitude of 45° , is 57028 French toises (see p. 327. Phil. Transf. 1768): and the same result nearly is obtained by taking a mean among all the measures of degrees there put down, that mean being 57038 toises. I shall therefore use the round number 57030 as probably nearer the truth. This number being multiplied by 6, the product 342180 shews the number of French feet in one degree; but, by p. 326. of the same volume, the lengths of the Paris and London feet are as 76.734 to 72, that is, as 4.263 to 4; therefore, as $4 : 4.263 :: 342180 : 364678$ = the English feet in one degree; and this being multiplied by 360 the whole number of degrees, there results

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131284080 feet for the whole circumference, which are equal to $24864\frac{1}{2}$ miles, making $69\frac{1}{15}$ to a degree in the mean latitude. Lastly, $\frac{2}{3}$ of 131284080 give 87522720 for the measure of the attraction of the whole earth.

Consequently, the whole attraction of the earth is to the sum of the two contrary attractions of the hill, as the number 87522720 to $8811\frac{2}{3}$, that is, as 9933 to 1 very nearly, on supposition that the density of the matter in the hill is equal to the mean density of that in the whole earth.

But the Astronomer Royal found, by his observations, that the sum of the deviations of the plumb line, produced by the two contrary attractions, was 11.6 seconds. From hence it is to be inferred, that the attraction of the earth is actually to the sum of the attractions of the hill, nearly as radius to the tangent of 11.6 seconds, that is, as 1 to .000056239, or as 17781 to 1; or as 17804 to 1 nearly, after allowing for the centrifugal force arising from the rotation of the earth about its axis.

Having now obtained the two results, namely; that which arises from the actual observations, and that belonging to the computation on the supposition of an equal density in the two bodies, the two proportions compared must give the ratio of their densities, which is

that of 17804 to 9933, or 1434 to 800 nearly, or almost as 9 to 5. And so much does the mean density of the earth exceed that of the hill.

Thus then we have at length obtained the object which we have been in quest of through the very laborious calculations that have been described in this paper, and in the survey and measurements from which these computations were made; namely, the ratio of the mean density of all the matter in the earth, in comparison with the density of the matter of which the hill is composed. And that ratio we have found to be equal to the ratio of 9 to 5. And, for the reasons before mentioned, I think we may rest satisfied, that this proportion is obtained to a considerable degree of proximity, probably to within the fiftieth part, if not the hundredth part of its true magnitude. Another question, however, still arises, namely, what is the density of the matter in the hill? Is its mean density equal to that of water, of sand, of clay, of chalk, of stone, or of some of the metals? For, according to the matter, or different sorts of matter, of which it is formed, and according as it is constituted with or without large vacuities, its mean density may be greater or less, and that in a degree which is not certainly known. A considerable degree of accuracy in this point could only be obtained by a close examination of the internal
structure

structure of the hill. And the easiest method of doing this would be to procure holes to be bored, in several parts of it, from the surface to a sufficient depth, after the manner that is practiced in boring holes to the coal mines from the surface of the ground; for by such operation it is known what kind of strata the borer is passed through, together with their dimensions and densities. The proper mean among all these would be the mean density of the hill, as compared to water or to any other simple matter; and thence we should obtain the comparative density of the whole earth with respect to water: but in the present instance, we must be satisfied with the estimate arising from the report of the external view of the hill; which is, that to all appearance it consists of an entire mass of solid rock. It is probable, therefore, that we shall not greatly err, if we assume the density of the hill equal to that of common stone; which is not much different from the mean density of the whole matter near the surface of the earth, to such depths as have actually been explored either by digging or boring. Now the density of common stone is to that of rain water as $2\frac{1}{2}$ to 1; which being compounded with the proportion of 9 to 5 above found, there results the ratio of $4\frac{1}{2}$ to 1 for the ratio of the densities of the earth and rain water; that

that is to say, the mean density of the whole earth is about $4\frac{1}{2}$ times the density of water.

To what useful purposes the knowledge of the mean density of the earth, as above found, may be applied, it is not my business here to shew. I shall therefore put an end to this paper with a reflection or two on the premises before delivered. Sir ISAAC NEWTON thought it probable, that the mean density of the earth might be five or six times as great as the density of water; and we have now found, by experiment, that it is very little less than what he had thought it to be: so much justness was even in the surmises of this wonderful man! Since then the mean density of the whole earth is about double that of the general matter near the surface, and within our reach, it follows, that there must be somewhere within the earth, towards the more central parts, great quantities of metals, or such like dense matter, to counterbalance the lighter materials, and produce such a considerable mean density. If we suppose, for instance, the density of metal to be 10, which is about a mean among the various kinds of it, the density of water being 1, it would require sixteen parts out of twenty-seven, or a little more than one-half of the matter in the whole earth, to be metal of this density, in order to compose a mass of such mean density as we have found the earth to possess by

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our experiment: or $\frac{4}{15}$, or between $\frac{1}{3}$ and $\frac{1}{4}$ of the whole magnitude will be metal; and consequently $\frac{20}{31}$, or nearly $\frac{2}{3}$ of the diameter of the earth, is the central or metalline part.

Knowing then the mean density of the earth in comparison with water, and the densities of all the planets relatively to the earth, we can now assign the proportions of the densities of all of them as compared to water, after the manner of a common table of specific gravities. And the numbers expressing their relative densities, in respect of water, will be as below, supposing the densities of the planets, as compared to each other, to be as laid down in Mr. DE LA LANDE's astronomy.

Water . . .	1
The Sun . .	$1\frac{2}{15}$
Mercury . .	$9\frac{1}{6}$
Venus . . .	$5\frac{18}{15}$
The earth . .	$4\frac{1}{2}$
Mars	$3\frac{2}{7}$
The Moon . .	$3\frac{1}{11}$
Jupiter . . .	$1\frac{1}{24}$
Saturn	$\frac{13}{32}$

Thus then we have brought to a conclusion the computation of this important experiment, and, it is hoped, with no inconsiderable degree of accuracy. But it is the
first

first experiment of the kind which has been so minutely and circumstantially treated; and first attempts are seldom so perfect and just as succeeding endeavours afterwards render them. And, besides, a frequent repetition of the same experiment, and a coincidence of results, afford that firm dependance on the conclusions and satisfaction to the mind, which can scarcely ever be had from a single trial, however carefully it may be executed. For those reasons it is to be wished, that the world may not rest satisfied barely with what has been done in this instance, but that they will repeat the experiment in other situations, and in other countries, with all the care and precision that it may be possible to give to it, till an uniformity of conclusions shall be found, sufficient to establish the point in question beyond any reasonable possibility of doubt. What has been already done in the present case will render any future repetition more easy and perfect. But improvements may be made, perhaps both in the mode of computation and in the survey; in the latter, especially, there certainly may. Some improvements of this kind I have hinted at in some parts of this paper, which with others I shall here collect together, that they may readily be seen in one point of view. They are principally these. Procure one base, or more if convenient, very accurately measured, in such situation, that

as many more points as possible in the survey may be seen from it. Assume as many principal or eminent points and objects as may be proper and convenient; and from each one of them measure the angles formed by all the rest that can be seen, both horizontal and vertical angles, and repeat these observations, if convenient, with the instrument varied or reversed, taking the means among the several quantities of each angle. Take then as many sections of the ground, and as far extended in all directions, as the time and circumstances will possibly admit. Of the sections, those that are horizontal or level are the best, as they require no calculation; procure therefore as many as possible of them. In vertical sections observe the vertical angles, not in the plane of the section, but at some other point of which the bearing is also taken from the beginning of the section line, and where the horizontal angles of the poles are taken, for the reasons before mentioned in p. 723. And it will be a still farther convenience if the section be made in such direction as to form a right angle with the line drawn to the point or station from whence the vertical angles of the poles are observed, as may be seen from what is said in p. 721. It might, perhaps, be proper to make some experiments on a valley instead of a hill, taking two observatories at the two opposite sides of it, both for the
greater

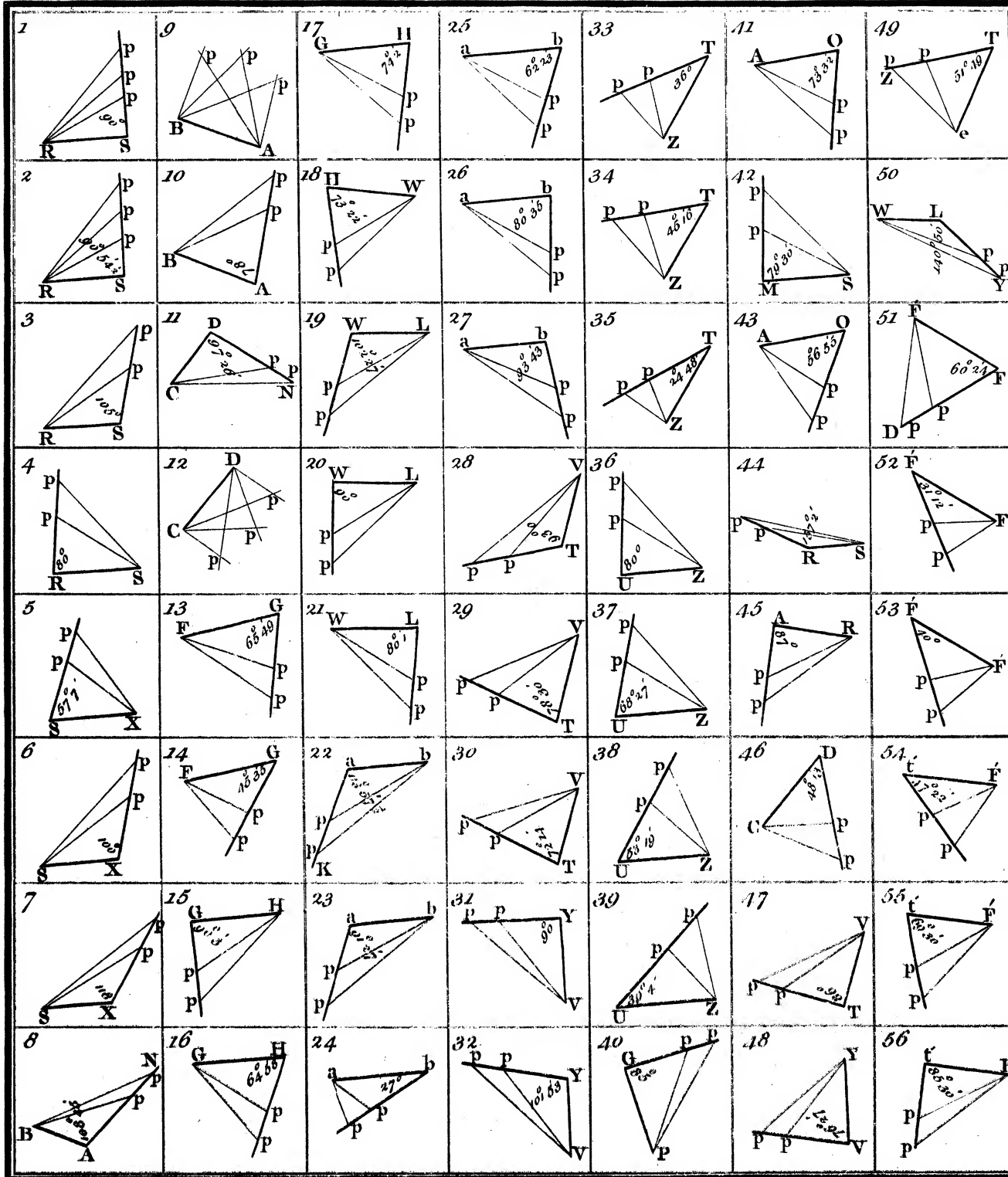
greater variety in this interesting problem, and because also the survey would be more easily made, on account of the ground being more in view at each station than in the case of a hill, which generally hides more than half the compass from the observer. In computing the relative altitudes of all the principal stations, let the operations be performed mutually both backwards and forwards, that is, from both of every two objects, having for that purpose observed at each of them the vertical angle of the other, namely, both the angle of elevation and the angle of depression, and take the mean between the two computed differences of altitude; for this excludes the necessity of making the proper allowances for refraction, and for the curvature of the earth; since the effect of each of these is balanced and corrected by that of the counter observation. But as to those points in the sections which are far distant from the observer, and where great accuracy is required, it may be proper to make the allowance for refraction and curvature, as there is generally no back observation by which their effects may be balanced. These are the chief hints which at present occur to me, besides the general information to be derived by the computer from the perusal of the modes of computation that have been described in this paper. As to the surveyor, he will strike out other

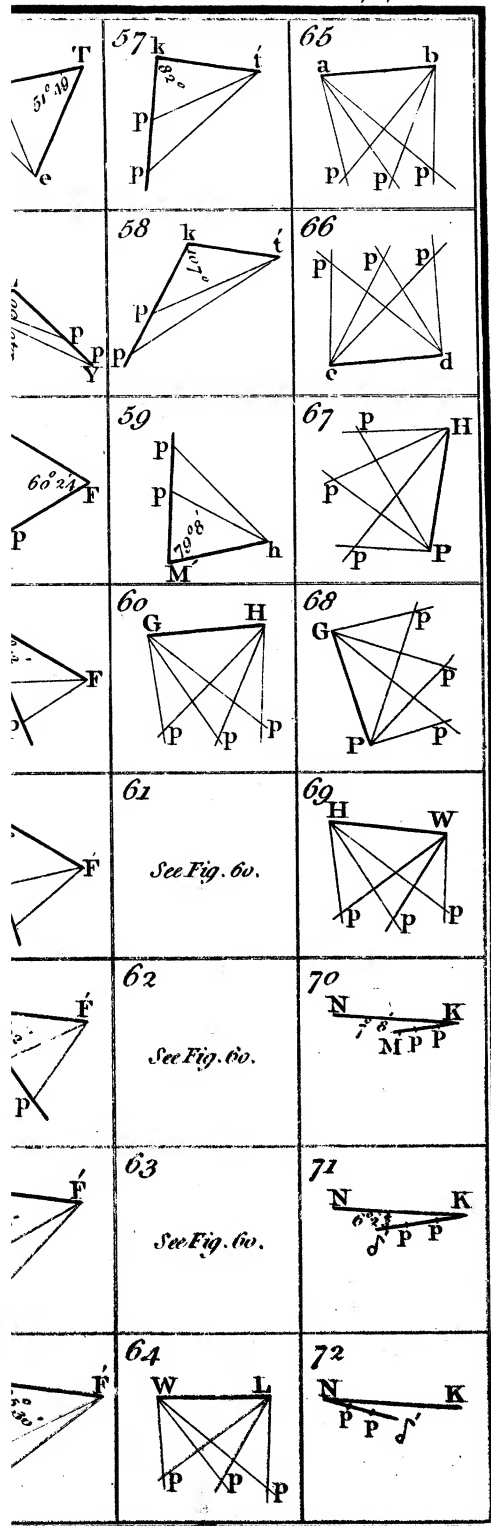
convenient ways of measurement adapted to the circumstances with which the nature of the survey may happen to be attended.

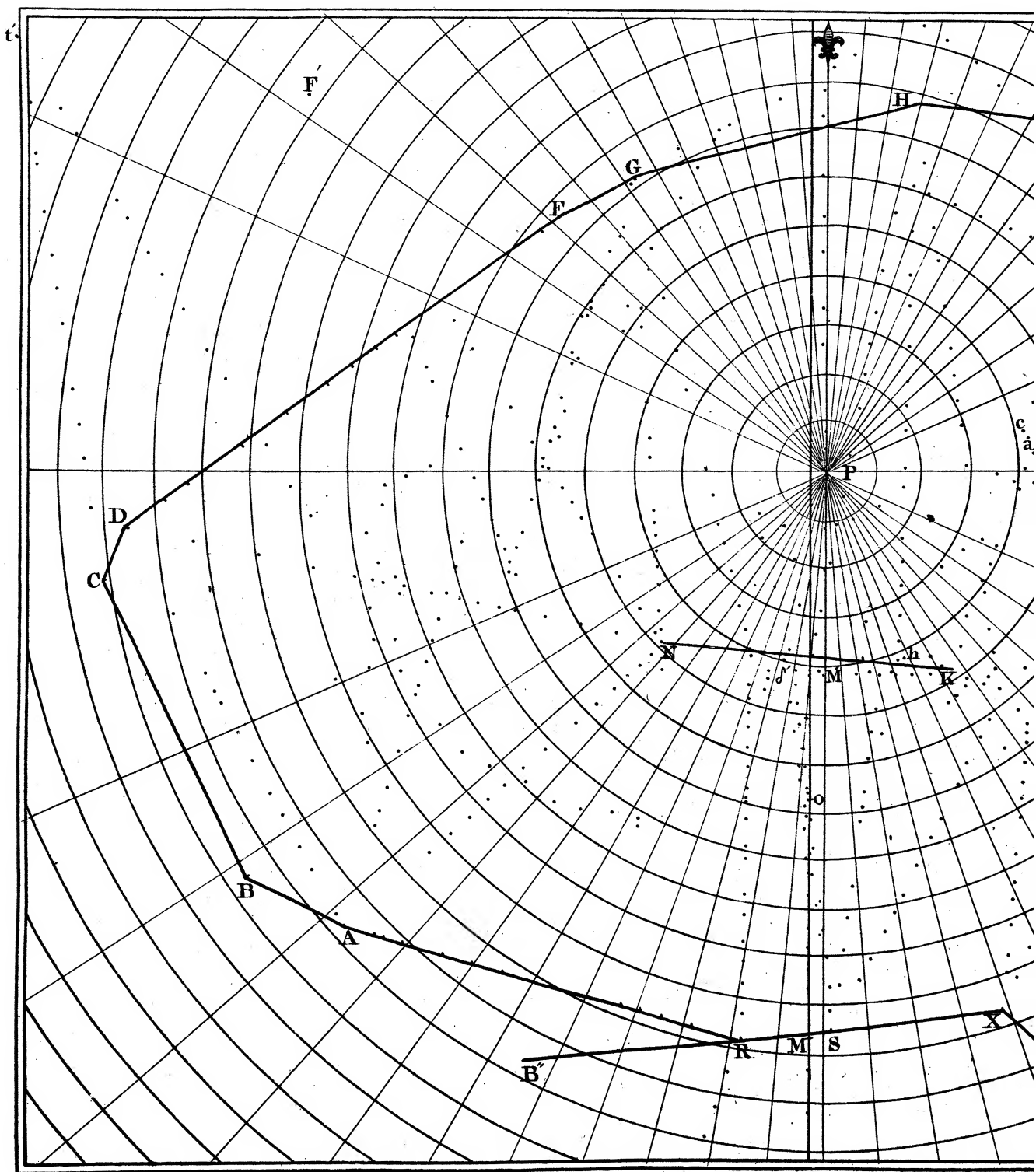
A map of the country about Schehallien is hereunto annexed, to convey a general idea of the nature of the ground, and for the better illustration of the description given in the former parts of this paper. This map is tab. XI.

Woolwich,
April 27, 1778.









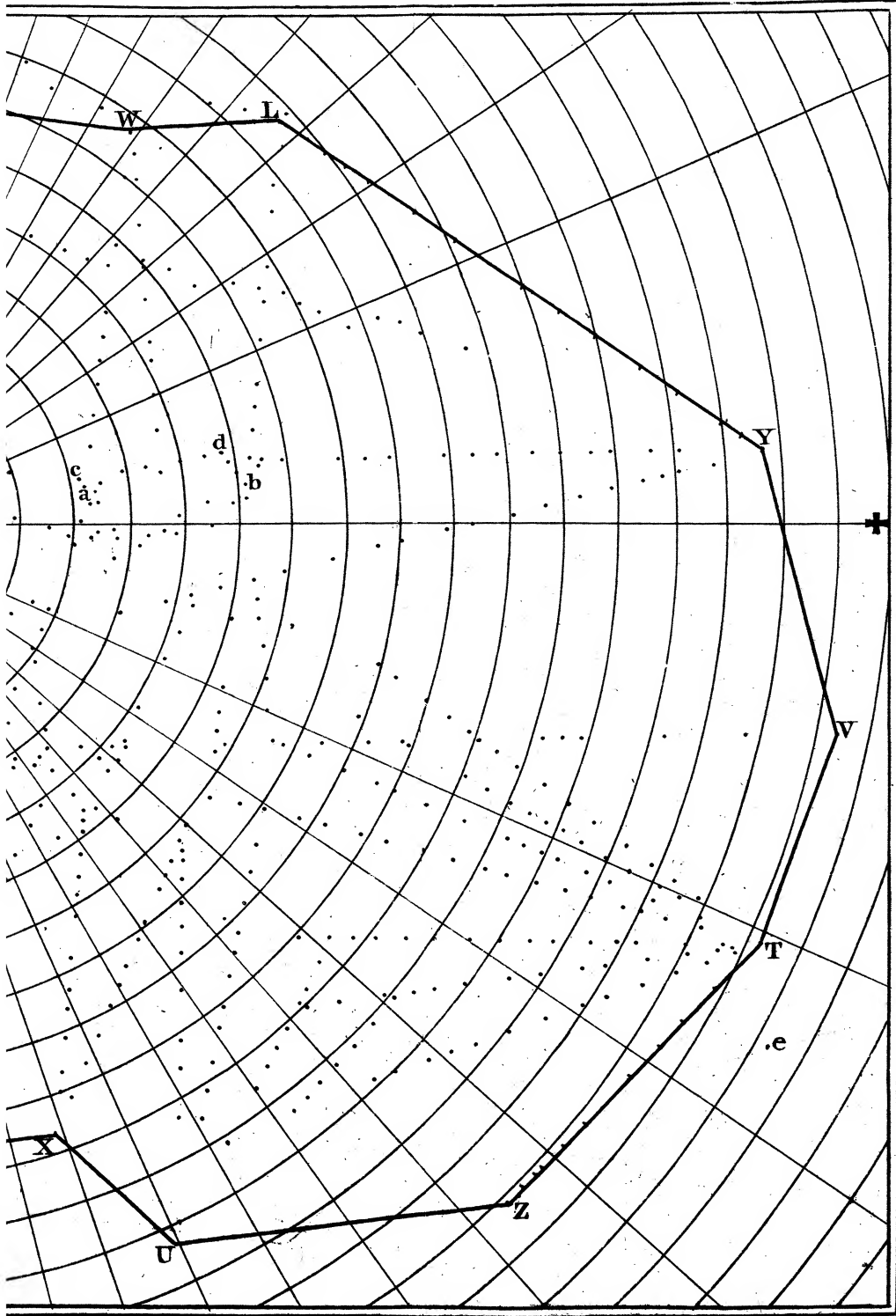


Fig. 1.

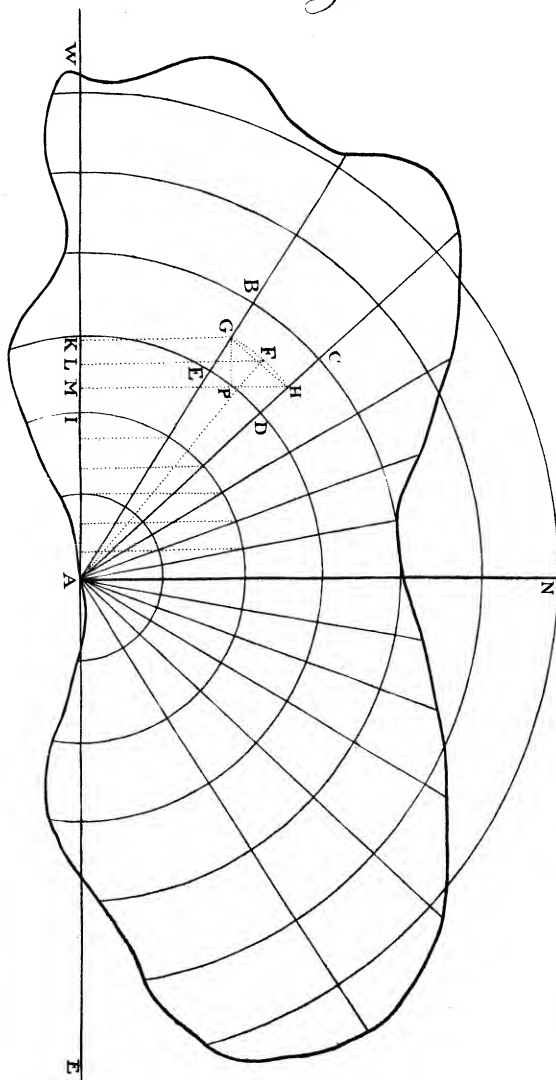


Fig. 2.

AF or Base	Perpendicular	Diff.
100	100	'001
106	106	'001
112	112	'001
119	119	'001
126	126	'001
133	133	'001
141	141	'001
150	150	'002
159	159	'002
168	168	'002
178	178	'003
188	188	'003
199	199	'004
211	211	'005
224	224	'005
237	237	'006
251	251	'008
266	266	'009
282	282	'011
299	299	'013
316	316	'015
335	335	'017
355	355	'020
376	376	'024
398	398	'028
422	422	'033
447	447	'039
473	473	'045
501	501	'053
531	531	'062
562	562	'072
596	596	'084
631	631	'097
668	668	'113
708	708	'130
750	750	'150
794	794	'172
842	842	'198
891	891	'226
944	944	'258
1000	1000	'293

